Extreme and Subjective Perspectives

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Abstract

This paper deals with ultra-wideangle perspectives and related curved perspectives. We distinguish between primary perspectives (referring to space situations when a perspective projection is applied) and secondary pseudo-perspectives that are induced later when a viewer looks at a perspective 2D-image.

In many cases, secondary perspectives induced by classic ultra-wideangle primary perspectives appear unrealistic and misleading. We are here discussing side effects of such projections. It is possible to overcome several drawbacks by means of a special curved perspectives. We are presenting two kinds of such perspectives: The first kind is generated by means of projecting space onto a sphere and called spherical perspective. The curved image is mapped onto 2D images, the transformed spherical perspectives. The second kind uses reflections on spheres (and cylinders of revolution). Both approaches are not linear anymore, i.e., straight lines will have curved images. We are giving some general rules for which approach is best for which purpose. The results are illustrated both by computer generated images and examples in art.

1 Primary and secondary perspectives

When we take a picture of a scene by means of a photographic camera, we have a projection of 3-space onto a flat image plane. The focus of the camera only has influence on the boundary (size) of the image: the smaller the focus, the more of the scene is visible. According to the rules of classic perspective, the images of straight lines appear as straight lines.

When we later on view the perspective image and move the eye point into the corresponding position, the situation in space can be more or less "reconstructed" by our brains. This works comparatively well as long as the new eye position does not differ too much from its requested position. Thus, there is a simple but important rule that should always be considered when creating a perspective image: *How will the viewer see the image?*

In the following, we will speak of the "primary" and the "secondary" perspectives: The first is the perspective at the moment when a picture is taken, the second is the pseudo-perspective, when a viewer looks at a flat perspective image from another viewpoint.

To give an example: The perspective in Figure 1 is created in the primary perspecticve by a camera with an ultra-wideangle lens (left). The focus of the lens (the eye point) is very close to the shark. In the middle of the first row of a cinema with a large screen the position of a viewer (secondary perspective) can be compared with the lens position. Thus, such a viewer will have a realistic – and very dramatic – impression of the situation (Figure 2). When the movie is shown on TV (a different secondary perspective), the relative viewing distance is maybe ten times larger. As a result, the viewer has the impression that the perspective is *very* unnatural.

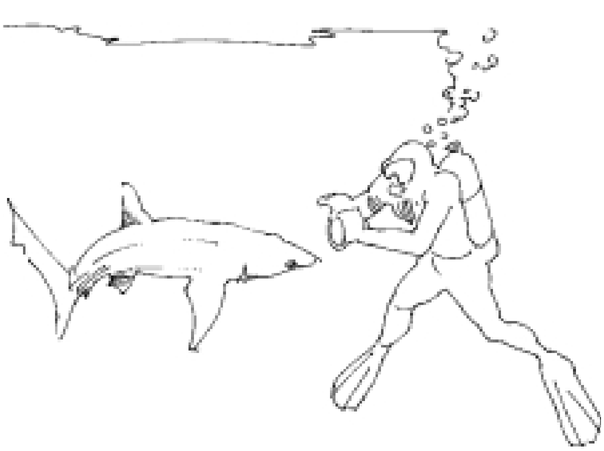

Fig. 1 Primary perspective projection

Fig. 2 Two secondary perspectives

In general we can say that classic primary perspective projections usually induce more or less realistic secondary projections for a focus $f \geq 30\,\mathrm{mm}$. The secondary perspectives become unrealistic and misleading for $18\,\mathrm{mm} < f < 30\,\mathrm{mm}$ and differ considerably from the images our brain normally produces with even smaller focuses: Distances and angles can then hardly be estimated, surfaces of revolution like spheres or cylinders appear unnatural, etc. (Figures 3, 19).

Again, everything depends on the viewer's position in regard to the 2D-image. When you view the images in Figure 3 separately from a very close distance (or – physically easier – you enlarge the images by e.g. factor ten without changing the eye position), your impression will be "normal":

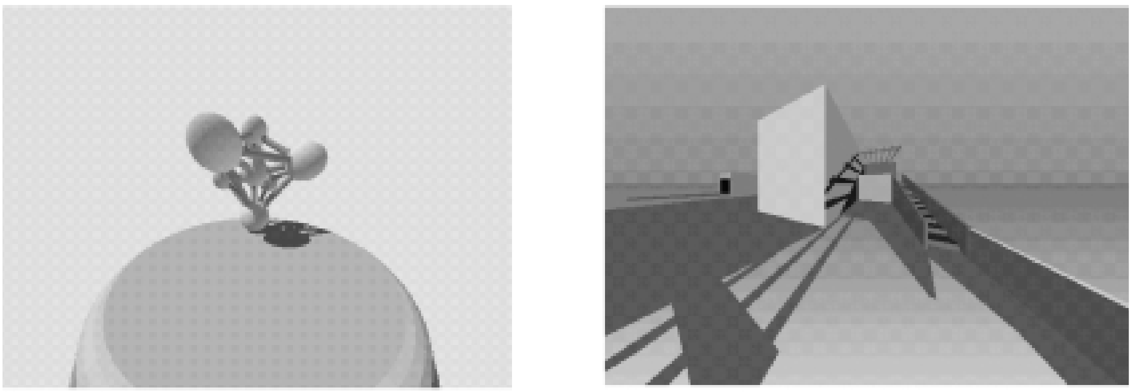

Figure 3: Typical images created by means of classic ultra-wideangle perspective.

Due to the close distance in the secondary perspective, the center of the image will appear larger, and everything is fine again. Thus, *a large figure in a magazine creates a different illusion than a small one* (Figure 4). Small figures require larger focuses (i.e., larger distances) in the *primary* stage!

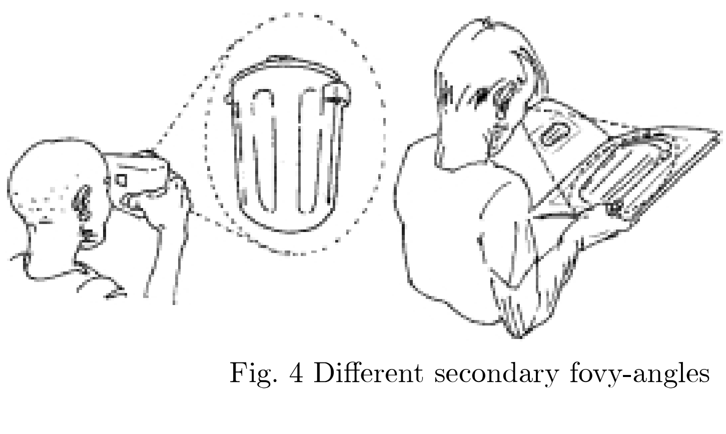

Fig. 4 Different secondary fovy-angles

2 The visual system

How does our visual system produce "images" in the primary stage? The human eye can only see sharply within a narrow cone with an apex angle ("field of vision angle" or fovy-angle) of about one degree (see Figure 5, [Bai59]). When we look at a scene that requires a large fovy-angle (i.e., a small focus) ([Hec74]), our eye balls will not stay fixed. Rather, they rotate quickly in order to produce several images of details of the scene ("centers of interest"). Our brain then unites those partial images into one "impression".

Fig. 5 The visible area

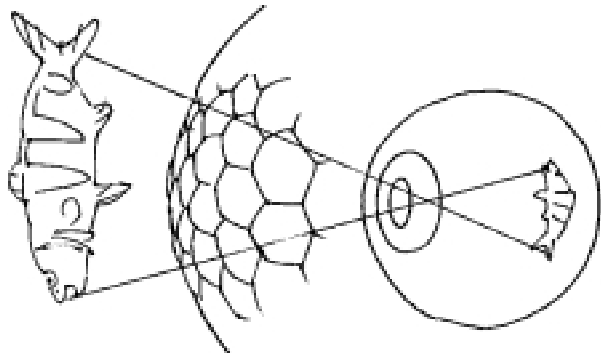

Fig. 6 Different eyes, similar results...

This impression only corresponds to images produced by classic primary perspectives with fovy-angles of less than 30° (see, e.g., [Str69]). Objects outside the cone of revolution with the eye point E as apex, the principal projection ray as axis and the apex angle of approximately 30° appear unrealistic, even in the primary perspective of the human visual system.

Whereas human eye balls can "roll" spherically, eyes of insects consist of hundreds (up to 30,000) of facettes, i.e. tiny cones. Each facette (*Ommatidium*) produces an image similar to the center of interest of the human eye. Thus, the sum of all part images is probably similar to the result of the rolling of the eye ball of higher developed animals (Figure 6).

The theory of the bad vision of insects is probably not true. Especially dragon flies, e.g., have an extremely good capability to spot their prey and to estimate distances. This is possibly the consequence of the spherical appearance of their eyes! Also, the so-called "momentum" is much higher: Wheras human eyes only need ≈ 20 images per second in order to consider a sequence of images as "movie", some insects need $4-5$ times as many images. Thus, a TV-movie is a rather slow slide-presentation for an ordinary mosquito...

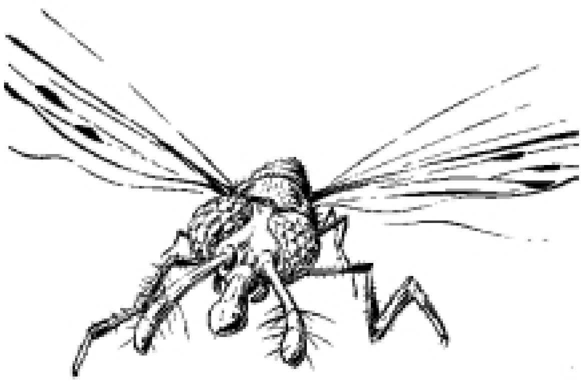

Fig. 7 Nightmare...

3 Distortions in ultra-wideangle perspectives

Now again back to images created by classic primary perspective ultra-wideangle projections. In a secondary perspective with “normal” eye position, the borders of the image are unnaturally enlarged. As a consequence, there is less space left for the center of interest, whereas details at the borders appear over-proportionally voluminous. When we take a picture of, e.g., a group of people, people’s faces at the borders of the picture are unnaturally distorted (Figure 8): People have “longer noses” and “bigger ears”, but also longer legs etc.

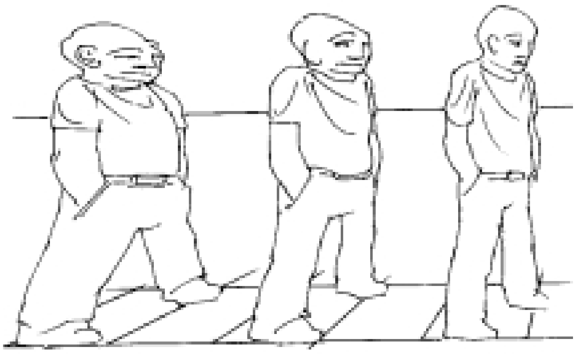

Fig. 8 Unwanted distortions

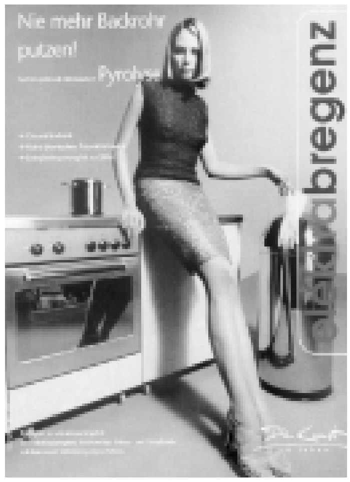

Fig. 9 Long legs. . .

The above mentioned drawbacks may turn into an advantage. Good photographers of course know about such distortions and they often use them for their purposes. Figure 9 exaggerates the long legs of the woman by means of an ultra-wideangle lens. The principal point of this photo is by far not the section of the diagonals (as it would be on the negative). It is rather somewhere to the left of the pot on the stove – not too far from the woman’s face, but very far from the woman’s lower legs. Note the mirrored image of the legs in the shield of the stove and also the curved reflection in the cylindric trash can. They both show the legs less distorted and closer to reality.

We usually expect that the silhouette of a sphere is a circle because when we observe a (not too large) sphere, we automatically and unconsciously rotate our eye ball(s) so that the principal projection ray approximately passes the center of the sphere. Then, the silhouette of the sphere is a circle even when we project 3-space onto a plane.

In classic perspective, however, the silhouette of a sphere may degenerate to an eccentric conic (Figure 3). The silhouette then encloses an area that is larger than it should be.

As a consequence, we will over-estimate the size of the sphere. Artists always knew about this fact. E.g., when Leonardo da Vinci painted his famous *Last Supper* (the building is an ultra-wideangle perspective, compare our scetch in Figure 10), he ignored the rules of perspective when it came to images of spheres (or human heads).

The above-mentioned drawbacks of classic perspectives can be diminished essentially with the help of appropriate curved perspectives. The price for this improvement, however, is that in general straight lines in 3-space will not appear as straight lines in the image. As people are used to see many photographic pictures every day, they are already used to the “fact” that

Figure 10: Leonardo paints his Last Supper.

straight lines have to appear as straight lines – though this is only true in classic perspectives and does not correspond to the way the human visual system perceives.

Wearer of contact lenses know well about the following phenomenon: When replacing the lenses by glasses, at the first moment space seems to appear "curved" (Figure 25). Seconds later, however, the brain "bends back" curved images of straight lines. This shows that the preliminary "hardware-result" of our visual system is manipulated by our brain to a "software-result".

4 Non-classic perspectives in art and computer graphics

Figure 11 shows that the human brain can also measure lengths by means of viewing angles. If that is so, then the rectangular wall in front of the viewer will of course not appear as rectangle – as it would in classic perpectives. Rather, the long horizontal borders will appear curved in the image. This was scientifically investigated first by G. Hauck ([Hau79]).

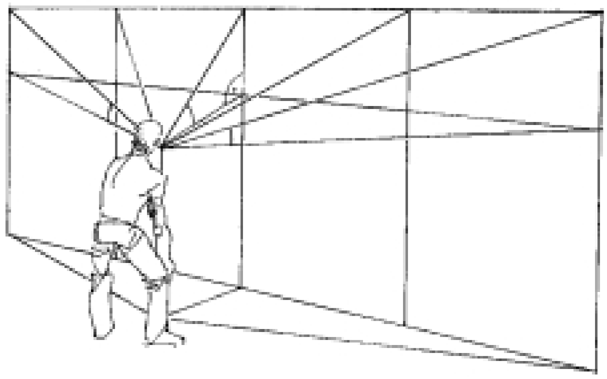

Fig. 11 Measuring lengths by angles

He suggested the projection onto a sphere (or a cylinder of revolution respectively) and then the interpretation of the spherical coordinates (i.e., the viewing angles) as two-dimensional coordinates.

Edgar Degas seemed to know about Hauck's ideas – or he used them intuitively. In his *Room at castle Ménil Hubert* (1892) (see Figure 12) one can clearly recognize curvatures that are like those in Hauck's subjective perspectives (e.g., watch the lower egdes of the paintings on the wall).

Of course, many artists made use of non-standard projections. M.C. Escher, e.g., worked with unusual projections, distorted views and distorted images.

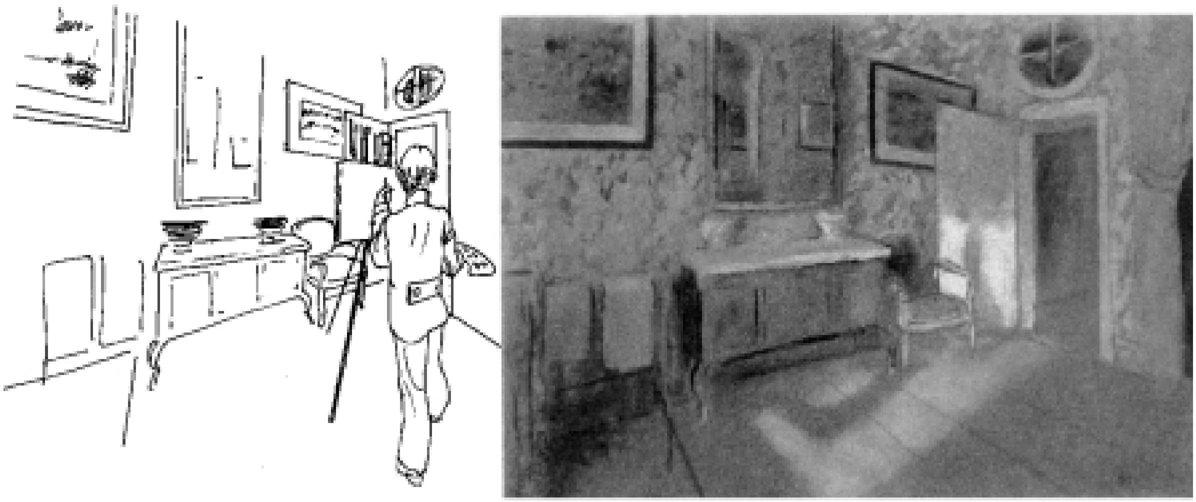

Figure 12: DEGAS' curved perspective. Left: the painter at work, right: the subjective result.

In computer graphics, non-classic perspectives are not very common. The reason for this is that the creation of classic perpectives via computer is usually supported by graphics hardware. Especially, clipping, polygon filling and depth buffering are essential for the generation of such images.

There have been approaches in computer graphics which model the human visual perception more accurately than perspective projection. A detailed survey over non-standard projections is given in [GG99]. Here we only mention two approaches: Cartographic projections [Pae90] map the surface of the earth onto 2D maps. Bayarri [Bay95] uses a cylindrical projection model for computing non-planar perspectives in real time. The approach is based on the assumption that images with a slight curvature give a more natural look to the scene.

Polack, Piegl and Carter [PPC97] investigate characteristics of the human visual system that affect the perception of computer generated images. They introduce a cylindrical projection system to create images which are closer to the one produced by the retina than a simple perspective projection. With actual psychological tests the size, shape, and relative depth of the perception are investigated. The tests confirm that with cylindrical projections an increased readability of simple line drawings is achieved.

Related work also includes investigations of map distortions. An example is the floating ring concept [BP98]. A circle is positioned interactively on a sphere and the corresponding projection of the circle is displayed to study area and angular distortions.

5 The spherical perspective

Classical perspective projects space from an eye point E onto a flat projection plane π (target point T). In order to avoid the disadvantages of ultra-wideangle lenses, we will now replace π by a projection sphere Σ around E (radius $\overline{ET}$). We will speak of the "spherical perspective" henceforth.

This is the simplest curved perspective and also the most natural one, since the human eye ball is very much like a sphere (see Figure 13). Therefore the relevant part of the retina follows spherical motions when we roll the eye ball quickly in order to “scan” 3-space to get a survey.

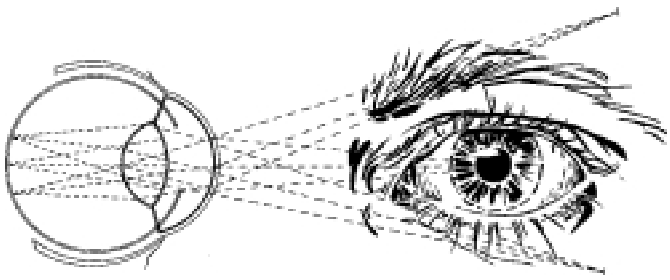

Fig. 13 The human eye ball

The creation of the spherical perspective works as follows: A point $P \in \mathbb{R}^3$ is projected by means of the projection ray EP on to a point $P^s \in \Sigma$ (actually, it is a pair of points, but we decide for the one closer to π). A straight line s is then projected onto the section line s^s of Σ by using the projecting plane Es, i.e. an arc of a great circle on Σ (again, we mean the arc closer to π). A circle in 3-space is transformed into the section line of the quadratic projection cone with Σ, i.e. a symmetrical oval spherical curve of degree 4, etc. The spherical perspective is thus non-linear (quadratic). The silhouette of an arbitrary sphere appears as the intersection of a cone of revolution with apex E with Σ, i.e., a small circle. This result corresponds with the fact that most people think that the silhouette of a sphere is a circle.

6 Curved perspectives derived from the spherical perspective

Unlike the classic perpective image, the spherical perspective image is non-planar. For practical applications we have to transform it into 2D (the screen or photographic paper). This can only be done by mappings – the sphere is not developable. All these mappings will be called “transformed spherical perspectives” henceforth. Of the various proposals that have been made so far (see, e.g., [Str69], [Havl98], Figure 14), we only mention four (a more detailed discussion can be found in [GG99]):

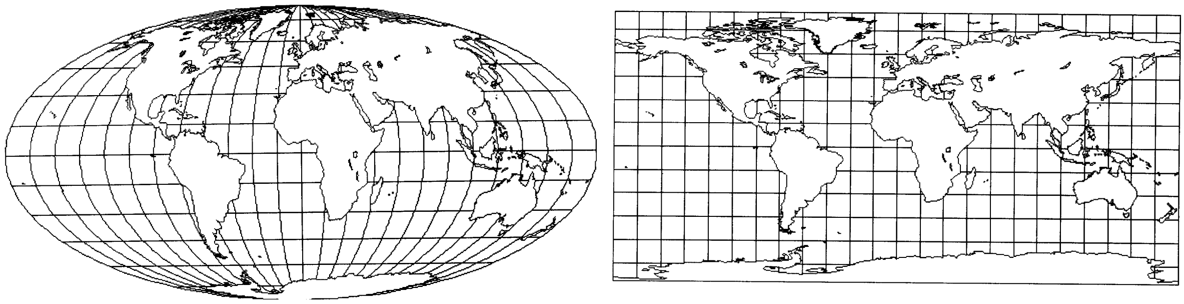

Figure 14: Mollweide mapping and Cylindrical projection (Hauck).

• Orthogonal projection: The image points $P^s \in \Sigma$ are projected orthogonally onto an image plane perpendicular to ET. This works unequivocally only for a fovy-angle $\varphi < 180°$. Lines (the spherical images of which are arcs of great circles) appear as arcs of ellipses. Such projections are good mainly for special effects, but do not really improve image quality.

• Stereographic projection ([Hoh66]): Let $D \in \Sigma$ be the point opposite T. We project the points $P^s \in \Sigma$ from the center D onto π. This projection turns out to be suitable for creating subjectively correct images.

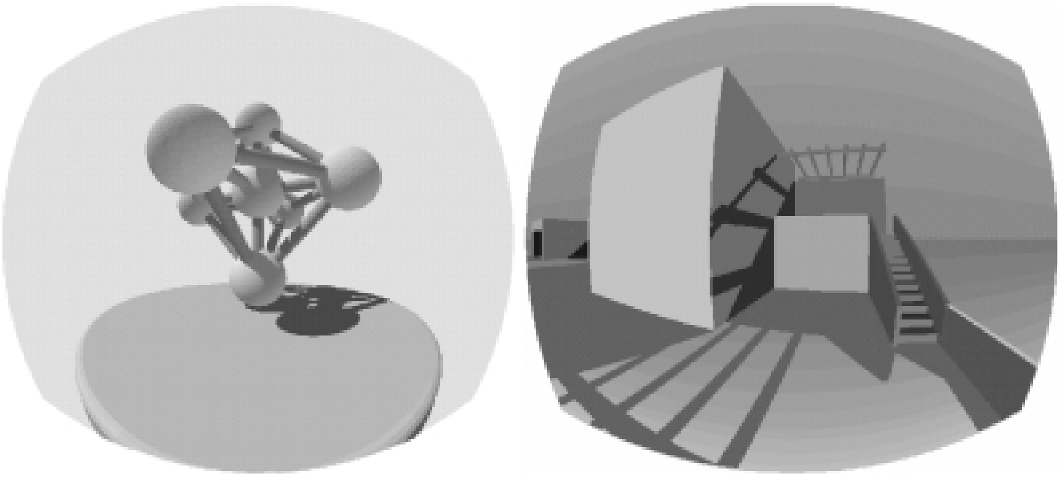

Figure 15: Typical images created by means of stereographic projection.

The images of straight lines are generally arcs of circles and thus have no point of inflection. Straight lines orthogonal to π (i.e., parallel to the main projection ray) appear as straight lines). A drawback is that vertical lines parallel to π (these lines are common in architectural photography) do not appear as straight lines. At least, images of straight lines do not have points of inflections.

The major advantage of the stereographic projection is that it preserves the property that the silhouette of a sphere is always a circle ([Hoh66]).

• Cylindrical mapping (HAUCK'S perspective): The spherical coordinates (azimuth angle λ and elevation angle μ) are interpreted as planar coordinates ($u = \lambda,\ v = \mu$). This mapping was suggested by G. HAUCK ([Hau10]) who was one of the pioneers in the geometrical investigation of "subjective perspective". When the principal projection ray is horizontal, vertical lines appear as vertical lines which seems to be important for human perception. On the other hand, the images of general straight lines are transcendental curves that may even have inflection points (this appears somewhat unnatural to the eye).

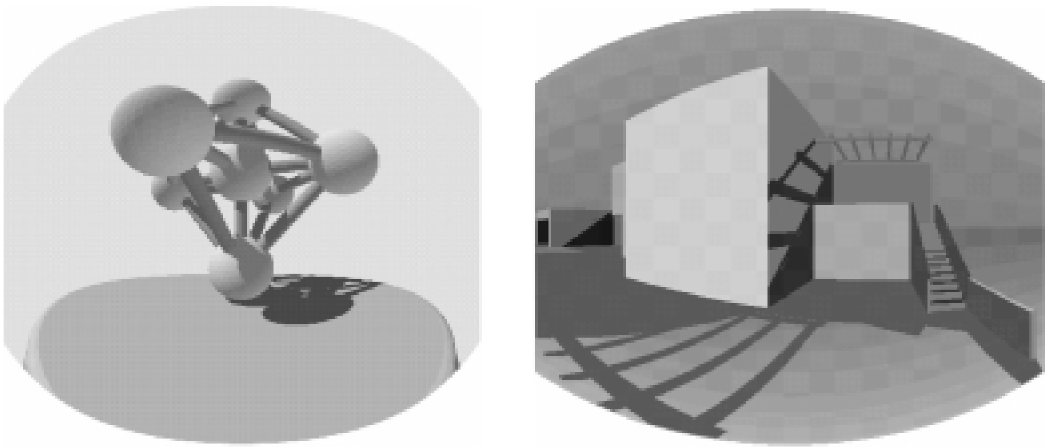

Figure 16: Typical images created by means of HAUCK's perspective.

Silhouettes of spheres are not circles but oval transcendental curves that look much like not too eccentric ellipses. When the spheres are comparatively small, however, the area enclosed by the oval silhouette comes close to the original circular area on the unit sphere. We may say that the mapping is *locally* area-preserving. This implies good distance- and volume-estimation for the viewer.

• Area-preserving mappings by MOLLWEIDE and ECKERT ([Str69]): For best distance- and volume-estimation, the mapping should be area-preserving *globally*. Among the numerous proposals for area-preserving mappings we chose the mappings by MOLLWEIDE and ECKERT, mainly because their results look natural (other mappings by LAMBERT, BONNE, ALBERS, STABIUS et.al. (see [Str69], [Havl98]) distort the scene unnaturally.

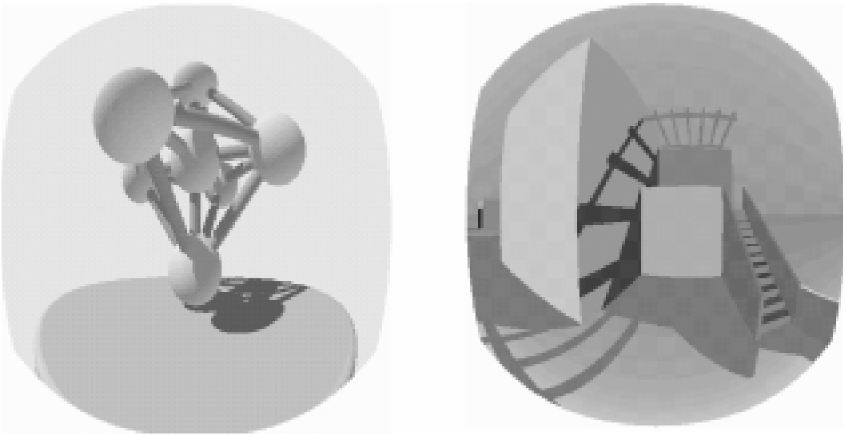

Figure 17: Typical images created by means of area preserving mappings (ECKERT).

Both mappings require the solution of a transcendental equation. These complex formulas produce transcendental results even when we simply map a straight line in space. The ECKERT mapping tends to produce lines with inflection points which look a little unusual.

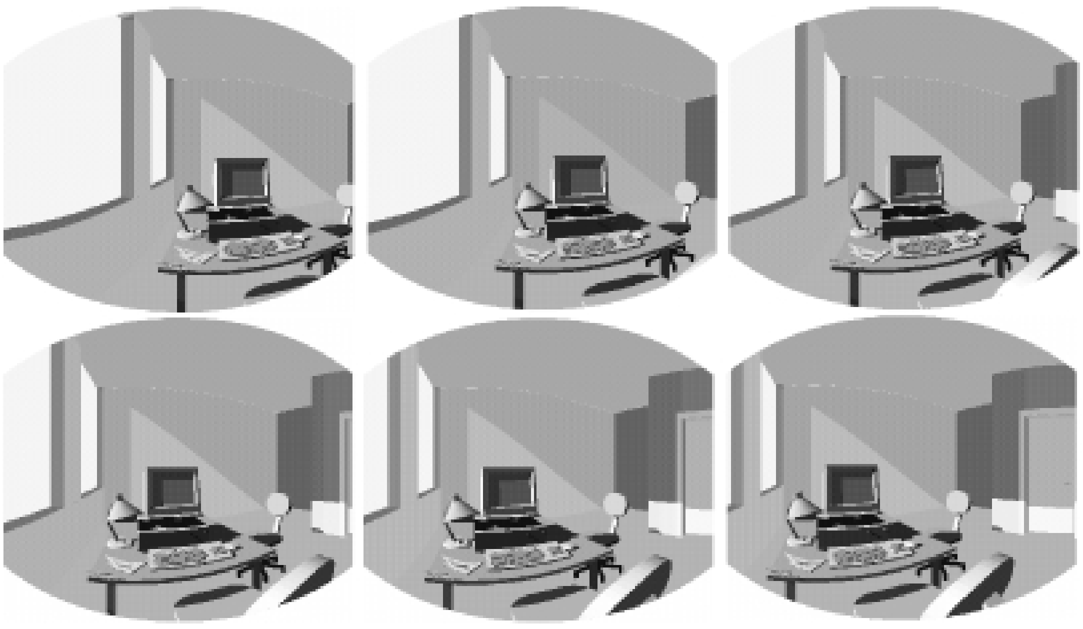

Figure 18: Virtual reality: Rotation of an ultra-wideangle camera (HAUCK's perspective)

In [GG99], the fast implementations of the above mentioned mappings are described. They build upon polygon-oriented methods developed in [Gla94]. Images like Figure 18 (ultra-wideangle camera in a small room) can be rendered in real time on an ordinary PC. Thus, such curved perspectives are appropriate for virtual reality applications.

In Figure 19, the same animated sequence is carried out by classic ultra-wideangle perspectives – with all their drawbacks (unnatural distortions, the center of interest appears too small, etc.).

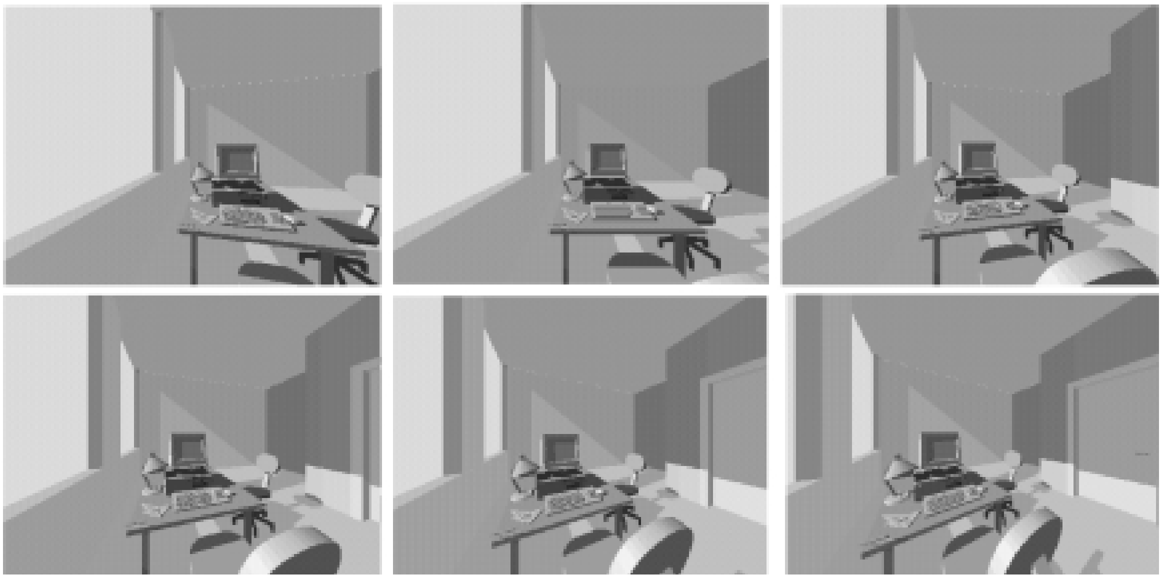

Figure 19: The same rotation of the camera, this time using classic perspectives

7 Reflections on spheres and cylinders of revolution

When it comes to the problem of "how to display as much as possible without diminishing the center of interest", we can also use reflections on curved surfaces. Reflecting spheres and cylinders of revolution turn out to be best since they produce the most "understandable" reflections that can be reconstructed by our brains without major difficulties. As an example, more or less spherical traffic mirrors are used to give the driver a better survey about the traffic situation (Figure 20). Cylindrical rear mirrors are sometimes used for the same purpose, in some countries with the writing "things in rear appear smaller than they are"...

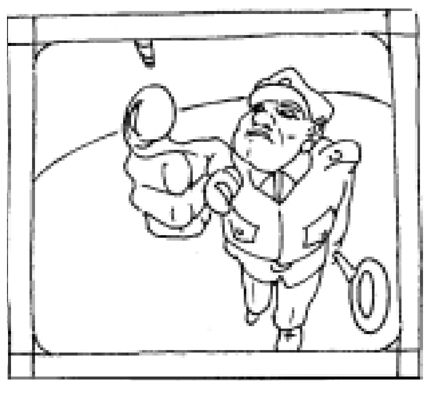

Fig. 20 Spherical traffic mirror

In architecture, reflections of the surrounding area often are an integral part of the optical appearance of the building. Reflections on planar surfaces are common, also reflections on *approximations to* curved surfaces. A classic example for a reflection on a *really* curved surface is H. Hollein's "Haas-Haus" (1987-90) in the center of Vienna (Figure 22). The perfect cylindrical shape of the front part allows the complete reflection of St. Stephen's cathedral (and other historical buildings) from several points of view. In fact, these reflections are always visible when walking around the building.

Fig. 21 Spherical reflections

Fig. 22 Haas-Haus

M.C. Escher is famous for his unusual projections ([Ern94]). Figure 21 shows a drawing that could be his... Again, the center of interest is comparatively little distorted, when we take into account that almost the whole room can be seen inside the contour of the reflecting sphere in the drawer's left hand.

Other examples can be found, e.g., in [Hof87]. Among others we want to mention F. Mazzolas *Self-portrait in convex mirror* (1523-24)and R. Hausners *Großer Laokoon* (1963-67).

Reflections on cylinders of revolution are described in [Elf81]. In [Gla99], such reflections are investigated in detail. The reflection of a point requires the solution of an algebraic equation of order four. Straight lines appear in general as curves of order four, circles as curves of order 8, etc.

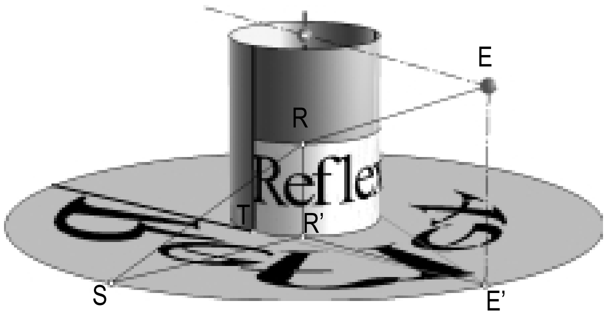

Fig. 23 Reflection at a cylinder

The reflected light (projection) rays envelope the so-called *catacaustic surfaces* (Figure 24, left). Thus, reflections are not simple distortions like the before-mentioned mappings of the sphere, and it happens frequently that one can see both sides of a polygon (Figure 24, middle and right).

8 Conclusion

To summarize: When we distinguish between primary and secondary classic perspectives (and we always should!), the simplified didactical suggestions about classic perspectives have to be seen in relation to other aspects. Everything depends on the *secondary* position of the viewer. Although usually non-recommendable, ultra-wideangle perspectives are subjectively correct and necessary

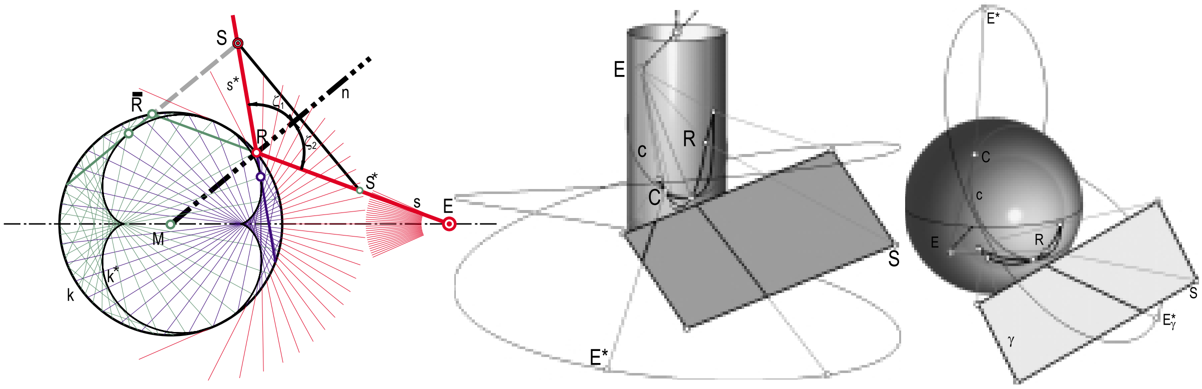

Figure 24: Katakaustic and “overlapping polygons”

when the viewer is forced into an extreme close position. On the other hand, perspectives with reasonably small field-of-vision angles seem exaggerated when the 2D-images are very small. They will also appear unnaturally distorted when the viewer is forced into an unusual position far away from the principal ray of the primary perspective.

Spherical perspectives are closer to the human visual system. 2D-mappings of such perspectives should be used instead of ultra-wideangle perspectives when the relative secondary distance is much longer than the primary one.

Depending on the displayed objects and the relative secondary distance, we recommend the following mappings: Stereographic projection of the spherical image for not too long distances and when spheres have to be displayed, Hauck's cylindrical mapping for normal to longer secondary distances and scenes with dominating verticals, area-preserving mappings for long secondary distances.

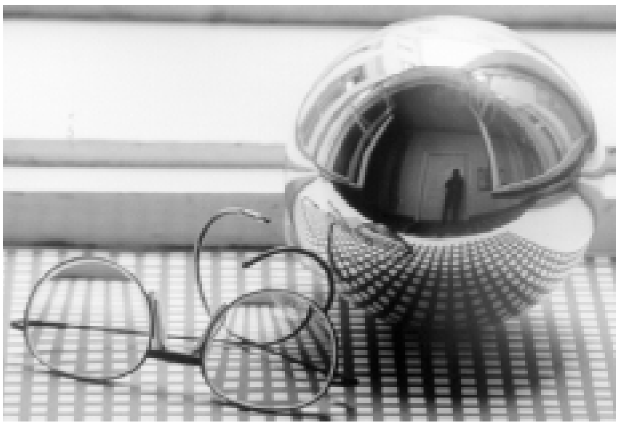

Figure 25: Reflections and refractions

Reflections (and partly also refractions) on spheres or cylinders are recommended as a good tool for displaying the viewer's complete environments in one image (Figure 25). Such images, however, require some experience for better understanding.

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