CURVATURE FUNCTIONS ON A ONE-SHEETEED HYPERBOLOID

Boris ODEHNAL
University of Applied Arts Vienna, Austria

ABSTRACT: We study the distribution of some curvature functions on a one-sheeted hyperboloid by determining, describing, and visualizing the curves of constant Gaussian, Mean, principal curvature, and the curves of constant ratio of the principal curvatures. Our aim is a precise description of the regions of prescribed curvature values. It turns out that all these curves are algebraic and can be given in terms of implicit equations. Surprisingly, it is possible to derive an explicit parametrization of the curves of constant principal curvature in terms of algebraic functions.

Keywords: One-sheeted hyperboloid, Gauss curvature, Mean curvature, principal curvature, constant curvature, support function, striction curve, ratio of principal curvatures, principal view.

1. INTRODUCTION

Some CAD-systems offer tools for *curvature analysis*. With these tools parts of surfaces can be textured with a color map of the Gaussian and Mean curvature. The primitives in a CAD system are usually approximated by some free-form surfaces. Thus, the curvature analysis sometimes tends to produce strange results. Symmetries of surfaces cause symmetries in the distribution of curvatures (cf. Figure 1). Unfortunately, these symmetries are not reproduced by the curvature analysis tools. We aim at a precise description of the distribution of some well-known curvature functions on a one-sheeted hyperboloid. The case of a hyperboloid of revolution is not treated here, since allmost all of the curves we are dealing with are parallel circles in this case. The respective curves on an ellipsoid are studied in [6].

In Section 2 we study the distribution of the Gauss curvature. Then, Section 3 is devoted to the curves of constant Mean curvature. Finally, in Section 4 we derive and investigate the curves of constant principal curvature together with the curves of constant ratio of the two principal curvatures. We derive an explicit parametrization of the curves of constant principal curvature in terms of algebraic functions. Note that these curves do not agree with the principal curvature lines.

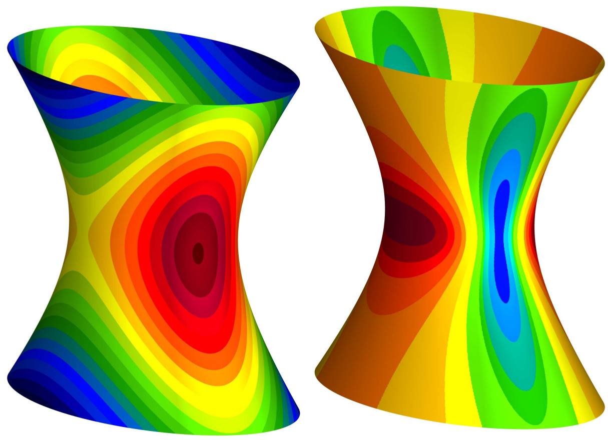

Figure 1: Gaussian curvature (left) and mean curvature (right) on a one-sheeted hyperboloid: Regions of a certain color correspond to curvature values within some interval.

In the following, when we describe the iso-curves of some curvature functions, we use the triplet of orthogonal projections onto the three mutually orthogonal planes of symmetry of the quadric. These planes shall coincide with the coordinate planes and we call the images appearing in the $[x,y]$-, $[y,z]$-, and $[x,z]$-plane the *top view*, the *front view*, and the *(right) side view*.

2. CONSTANT GAUSSIAN CURVATURE

The surface $\mathscr{S}$ in question shall be the one-sheeted hyperboloid with the equation

$$\mathscr{S}: \frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1 \tag{1}$$

where $0<a<b$ and $c>0$ are real coefficients. It is well-known that $\mathscr{S}$ can be considered as a ruled surface in two different ways: Both ruled surfaces

$$R_{1,2}(u,v)=\begin{pmatrix} a\cos u \\ b\sin u \\ 0 \end{pmatrix}+v\cdot\begin{pmatrix} -a\sin u \\ b\cos u \\ \pm c \end{pmatrix} \tag{2}$$

(with parameters $u\in[0,2\pi[$ and $v\in\mathbb{R}$) are entirely contained in $\mathscr{S}$.

Solving Eq. (1) for z and parametrizing $\mathscr{S}$ by $(x,y,\pm z(x,y))^{\mathrm{T}}$, the Gaussian curvature of $\mathscr{S}$ can be expressed in terms of the underlying Cartesian coordinate system as

$$K=-\frac{1}{a^2b^2c^2}\cdot\frac{1}{\left(\frac{x^2}{a^4}+\frac{y^2}{b^4}+\frac{z^2}{c^4}\right)^2}. \tag{3}$$

The *support function d* of $\mathscr{S}$, *i.e.*, the distance of $\mathscr{S}$'s tangent planes to the origin of the coordinate system is given by

$$\frac{1}{d}=\sqrt{\frac{x^2}{a^4}+\frac{y^2}{b^4}+\frac{z^2}{c^4}} \tag{4}$$

provided that the point of contact is the point $(x,y,z)^{\mathrm{T}}$ whose coordinates satisfy Eq. (1). Thus, we rewrite the formula for the Gaussian curvature given in Eq. (3) in terms of the support function of the one-sheeted hyperboloid and find

$$K=-\frac{d^4}{a^2b^2c^2}. \tag{5}$$

This is the analogue to a formula given by Wunderlich in [6] for the Gaussian curvature of an ellipsoid. Actually, this simple formula relating the distance of the tangent planes and the Gaussian curvature is derived for quadrics with center (including ellipsoids, one-, and two-sheeted hyperboloids) in [4]. However, only for the one-sheeted hyperboloids there is a negative sign.

The support function of $\mathscr{S}$ equals the distance of the surfaces points of $\mathscr{S}$ to the origin exactly at the vertices $(\pm a,0,0)^{\mathrm{T}}$ and $(0,\pm b,0)^{\mathrm{T}}$. Inserting $d=\pm a$ and $d=\pm b$ in Eq. (5) we find $K_a=-a^2b^{-2}c^{-2}$ and $K_b=-b^2a^{-2}c^{-2}$. Since $a<b$ by assumption we can easily recognize that the minimum of the Gaussian curvature on $\mathscr{S}$ equals $K_{\min}=K_b$ which is the case at the two vertices $(0,\pm b,0)^{\mathrm{T}}$.

The iso-curve with $K=K_a$ splits into a pair of congruent ellipses concentric with the quadric $\mathscr{S}$ (see Figure 2) with carrier planes through the x-axis. The length of its semi-minor axis equals a (*i.e.*, the semi-minor axis of $\mathscr{S}$ in the x-axis). The length of the semi-major axis of theses two ellipses equals $\frac{1}{a}\sqrt{a^2b^2+b^2c^2-c^2a^2}$.

Since a, b, c are constant and with Eq. (5) in mind, we can state:

Theorem 2.1. *The tangent planes of the hyperboloid $\mathscr{S}$ along a curve of constant Gaussian curvature K_0 are at fixed distance $d_0=\sqrt{abc}\sqrt[4]{-K_0}$ from the origin, and thus, they envelope a sphere which is concentric with $\mathscr{S}$ and has radius d_0.*

Independent from the choice of a, b, and c, K is always negative, as it was to be expected for a ruled surface without singular surface points. From Eq. (3) we can deduce:

Theorem 2.2. *The curves of constant Gaussian curvature on a one-sheeted hyperboloid with Eq.* (1) *are the quartic curves of intersection of the hyperboloid with concentric and coaxial ellipsoids $\mathscr{E}$ with equation*

$$\mathscr{E}: \frac{x^2}{a^4}+\frac{y^2}{b^4}+\frac{z^2}{c^4}=\frac{1}{abc\sqrt{-K}}=\frac{1}{d^2}. \tag{6}$$

Figure 2 shows some curves of constant Gaussian curvature on a one-sheeted hyperboloid.

Since $K<0$ for all points on $\mathscr{S}$, Eqs. (6) are the equations of ellipsoids with real points. We can reformulate this result as:

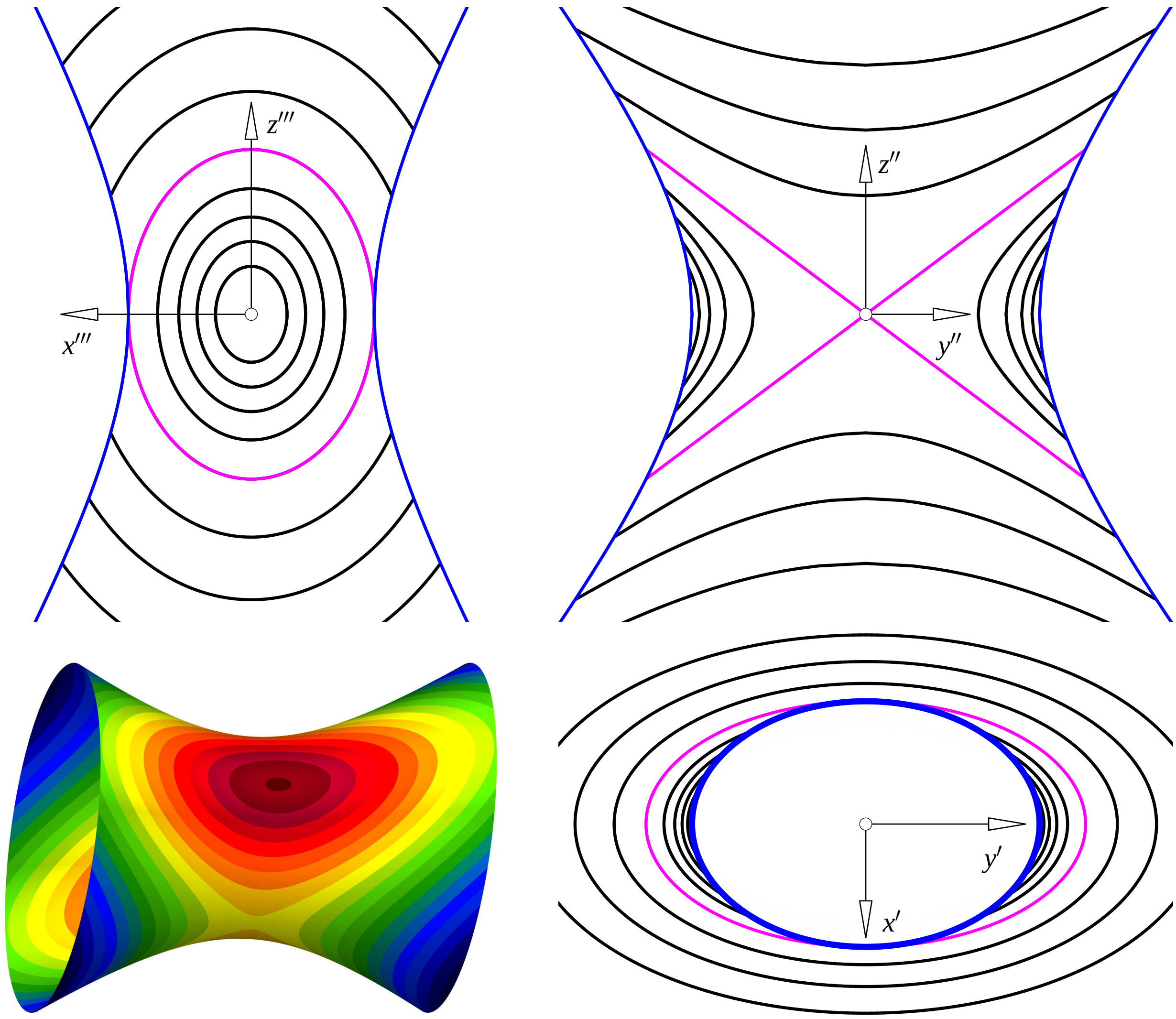

Figure 2: Curves of constant Gauss curvature: right side view and front view (top row, left and right), top view (below, right). The magenta ellipses are the iso-curves $K = K_a$.

Theorem 2.3. *The curves of constant Gaussian curvature $K_0 < 0$ on a one-sheeted hyperboloid are the curves of contact of a developable ruled surface tangent to the hyperboloid $\mathscr{S}$ with Eq.* (1) *and a concentric sphere of radius $d_0 = 1/\sqrt{abc\sqrt{-K_0}}$.*

This result is similar to that given by Wunderlich in [6] for the curves of constant Gaussian curvature on an ellipsoid.

Figure 3 illustrates the contents of Theorems 2.1 and 2.3. The part of the developable surface joining the curve of constant Gaussian curvature on $\mathscr{S}$ and a spherical curve (on the sphere $\mathscr{E}$ mentioning in Theorem 2.1) is shown.

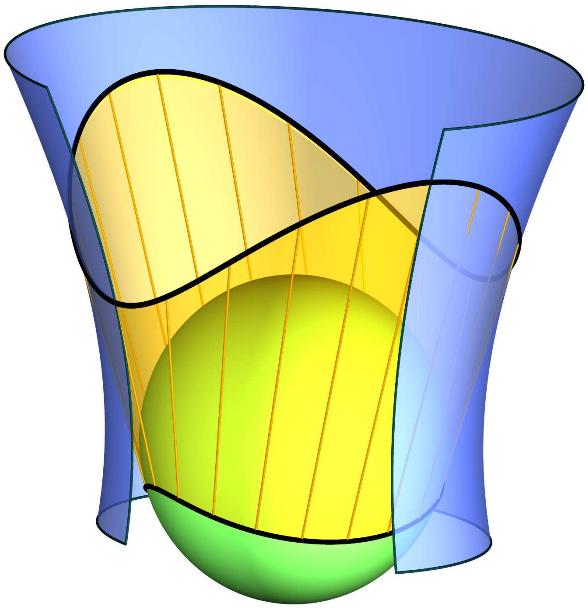

Figure 3: The tangent planes of $\mathscr{S}$ along a curve a constant Gaussian curvature are tangent to a concentric sphere and and form a developable surface which is in line contact with both quadrics.

Assume $\mathscr{S}$ is parametrized as one of the ruled surfaces given by Eq. (2) with directrix $l(u) = (a\cos u, b\sin u, 0)$ and the unit vector field $g(u)$ parallel to either $\overline{g} = (-a\sin u, b\cos u \pm c)$. (The $+$ and $-$ correspond to the *right* and *left* regulus.) According to LARMARLE's formula (see for example [2, 4]), the Gaussian curvature can be computed via

$$K = -\frac{\delta^2}{(\delta^2+v^2)^2} \qquad (7)$$

where $\delta = \delta(u) = \det(\dot{g}, g, \dot{l})\langle \dot{g}, \dot{g}\rangle^{-1}$ is the distribution parameter of the ruling $r(u) = d(u) + v \cdot g(u)$ and the parameter v equals the distance of the surface point (u, v) from the striction point. (The $\dot{}$ indicates differentiation with respect to u.) Obviously, the Gaussian curvature considered as a function on a ruling (or equivalently $K(u, v)$ restricted to $u = u_0$ (fixed)) attains its minimum exactly at $v = 0$ which corresponds to the striction point. Therefore, we can say:

Theorem 2.4. *The curves of constant Gauss curvature touch the rulings exactly at the striction points.*

Figure 4 shows that the rulings and the isocurves of K are in contact at the striction points.

For a fixed value $K_0 < 0$ the hyperboloid $\mathscr{S}$ (1) and the ellipsoid $\mathscr{E}$ from (6) span a pencil of quadrics passing through the curve with constant Gaussian curvature K_0. Within this pencil we find four singular quadrics. The first of which is a quadratic cone emanating from $(0,0,0)^{\mathrm{T}}$). The remaining three are

$$\begin{aligned} \mathscr{T}: &\quad b^4\beta\, x^2 + a^4\alpha\, y^2 = a^4b^4(c^2\lambda+1), \\ \mathscr{F}: &\quad c^4\gamma\, y^2 + b^4\beta\, z^2 = b^4c^4(a^2\lambda-1), \qquad (8) \\ \mathscr{R}: &\quad -c^4\gamma\, x^2 + a^4\alpha\, z^2 = a^4c^4(b^2\lambda-1), \end{aligned}$$

with $\alpha := b^2+c^2$, $\beta := c^2+a^2$, $\gamma := a^2-b^2$, and $\lambda^{-1} := abc\sqrt{-K}$. The equations of $\mathscr{T}$, $\mathscr{F}$, and $\mathscr{R}$ as given in (8) are the equations of the top, front, and right side view since they are relations in two variables only. From that we learn:

Theorem 2.5. *The curves of constant Gauss curvature on a one-sheeted hyperboloid are non-rational quartic curves.*

The top view and the right side view of the curves of constant Gauss curvature on a one-sheeted hyperboloid are ellipses. The front view of the curves of constant Gauss curvature on a one-sheeted hyperboloid are hyperbolae.

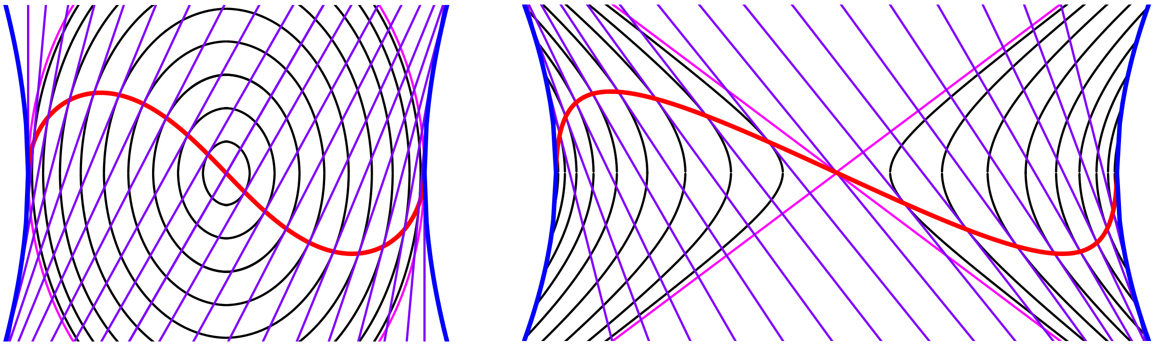

Figure 4: The curves of constant Gaussian curvature (black) touch the generators (violet) exactly at the central points. The central curve (striction curve) is shown in red (right-side view and front view).

Figure 2 shows the three principal views of the iso-curves of the Gaussian curvature. In Figure 4 the right side view and the front view are shown.

3. MEAN CURVATURE

The Mean curvature can be given in two equivalent ways: First, we can start with the parametrization $(x,y,z(x,y))^{\mathrm{T}}$ with z being a solution of Eq. (1). Thus, we have

$$M = \frac{d^3}{2a^2b^2c^2}L \tag{9}$$

with L being a quadratic funtion in x, y, z:

$$L=(b^2-c^2)\frac{x^2}{a^2}+(a^2-c^2)\frac{y^2}{b^2}-(a^2+b^2)\frac{z^2}{c^2}. \tag{10}$$

From Eq. (9) we can immediately see: The curves on $\mathscr{S}$ with vanishing Mean curvature $M=0$ are described by Eq. (1) and (10). The latter equation is that of a quadratic cone $\mathscr{L}$ emanating from $\mathscr{S}$'s center. Eliminating x, or y, or z, we obtain the equations of the top view $\mathscr{T}$, the front view $\mathscr{F}$, and the right side view $\mathscr{R}$ of the curve of vanishing Mean curvature. Thus, we have

$$\begin{aligned} \mathscr{T}:&\ b^2\beta\, x^2 \ +a^2\alpha\, y^2=a^2b^2(a^2+b^2),\\ \mathscr{F}:&-c^2\gamma\, y^2 \ +b^2\beta\, z^2=b^2c^2(c^2-b^2),\\ \mathscr{R}:&-c^2\gamma\, x^2 \ -a^2\alpha\, z^2=a^2c^2(c^2-a^2). \end{aligned} \tag{11}$$

Now we can show the following result:

Theorem 3.1. *The curve m of vanishing Mean curvature on the one-sheeted hyperboloid $\mathscr{S}$ with Eq.* (1) *splits into the pair of smallest circles on $\mathscr{S}$ if, and only if, $c=a$.*

Proof. By assumption $a<b$. If $c=a$, the right side view $\mathscr{R}$ given in Eq. (11) splits into ac pair of line segments on the lines $x\sqrt{b^2-a^2}\pm z\sqrt{a^2+b^2}=0$. These lines are the views of the projecting planes ζ and $\overline{\zeta}$ with the same equation and they are real. These two planes meet $\mathscr{S}$ in a pair of real ellipses. The semi-major and semi-minor axis of the ellipse m' showing up in the top view $\mathscr{T}$ are of length $a'=\sqrt{\frac{a^2+b^2}{2}}$ and $b'=b$. The right side view m''' is also an ellipse and its semi-major and semi-minor axis are of length $b'''=b$ and $a'''=\sqrt{\frac{b^2-a^2}{2}}$. Since $(a')^2+(a''')^2=b$, the second principal axis of m is of length b. This shows that m is a circle. All circles on $\mathscr{S}$ are contained in planes parallel to ζ and $\overline{\zeta}$. Since ζ and $\overline{\zeta}$ pass through the $\mathscr{S}$'s center and the vertices $(0,\pm b,0)^{\mathrm{T}}$, these two circles are the smallest on $\mathscr{S}$.

In case of $c=b$ the front view $\mathscr{F}$ given in Eq. (11) equals $y^2(b^2-a^2)+z^2(a^2+b^2)=0$ which is a pair of complex conjugate lines (or planes) since $a<b$. Thus the curve m is the intersection of a pair of complex conjugate planes with $\mathscr{S}$ and carries no real point. □

In a completely different way Krames has shown in [3] the following: If one point on the

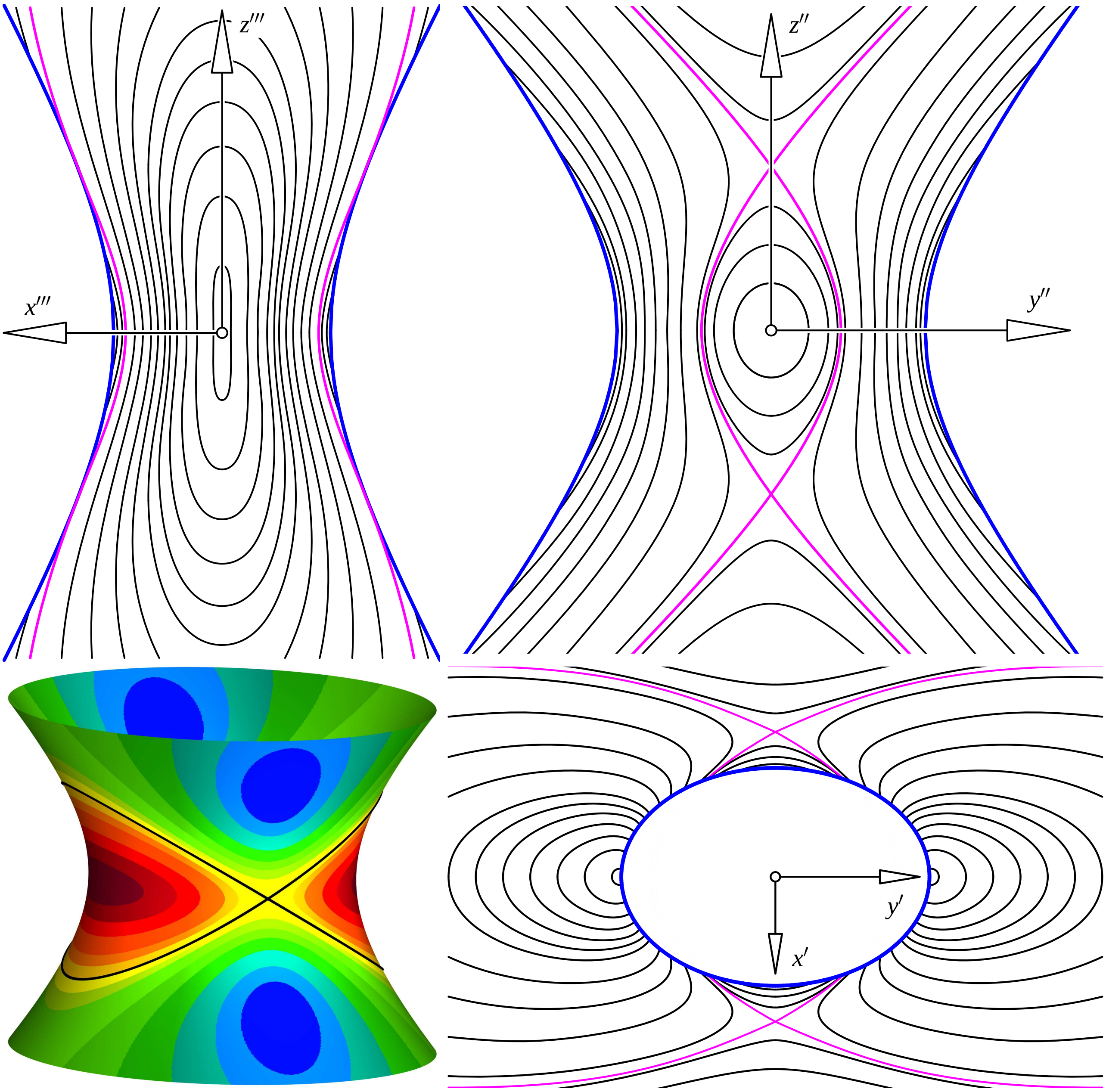

Figure 5: Curves of constant Mean curvature: right side view, top view, front view (top row and bottom row, right) showing the singular curve (magenta) on the hyperboloid $a = 1$, $b = \sqrt{2}$, $c = \sqrt{3}$; areas with positive (red) and negative (blue) Mean curvature separated by the (black) ellipses all of whose points show $M = 0$ on the hyperboloid $a = c = 1$, $b = \sqrt{2}$ (bottom, left).

smallest circles of a one-sheeted hyperboloid is a point of vanishing Mean curvature, then any point on the smallest circles is a point of vanishing Mean curvature.

Figure 5 (bottom row, right) shows a one-sheeted hyperboloid (with $a = c = 1$, $b = \sqrt{2}$) with its curve of vanishing Mean curvature consisting of a pair of circles.

In order to describe the curves of constant Mean curvature we can turn to an expression equivalent to Eq. (9)

$$4a^4b^4c^4M^2 = L^2d^6. \tag{12}$$

From Eq. (12) we can deduce:

Theorem 3.2. *The curves of constant Mean curvature on a one-sheeted hyperboloid are algebraic curves of degree* 12.

The principal views of the curves of constant Mean curvature on a one-sheeted hyperboloid are algebraic curves of degree 6.

Proof. The curves of constant Mean curvature on $\mathscr{S}$ are described by the quadratic equation (1) of $\mathscr{S}$ and the equation of a sextic surface with equation (12). According to Bezout's theorem the curves' degree equals $2 \cdot 6 = 12$.

Each of the prinicpal views is traced twice because of $\mathscr{S}$'s symmetry. Hence, the degree of the curves appearing in the three principal views reduces to $12 : 2 = 6$. $\square$

Among the curves of constant Mean curvature there are singular curves if the Mean curvature is either $\frac{bc}{3a^3\sqrt{3}}$ or $\frac{ac}{3b^3\sqrt{3}}$. These singular curves have four additional real double points at

$$\left(0, \pm\frac{b}{\sqrt{\alpha}}\sqrt{3a^2+b^2}, \pm\frac{c}{\sqrt{\alpha}}\sqrt{3a^2-c^2}\right)^{\mathrm{T}}$$

or

$$\left(\pm\frac{a}{\sqrt{\beta}}\sqrt{a^2+3b^2}, 0, \pm\frac{c}{\sqrt{\beta}}\sqrt{3b^2-c^2}\right)^{\mathrm{T}}$$

depending on whether $3a^2 - c^2 > 0$ or $3b^2 - c^2 > 0$. These curves are shown in Figure 5 (magenta curves).

4. PRINCIPAL CURVATURES

The principal curvatures κ_1 and κ_2 are the eigenvalues of the Weingarten mapping ω, cf. [1, 5]. If the Weingarten mapping on $\mathscr{S}$ is described by the quadratic matrix W, then the Gaussian curvature equals $K = \det W = \kappa_1\kappa_2$ and the Mean curvature equals $M = \frac{1}{2}\mathrm{tr}(W) = \frac{1}{2}(\kappa_1 + \kappa_2)$. From these two equations we can eliminate either κ_1 or κ_2 and find

$$\kappa^2 - 2M\kappa + K = 0 \tag{13}$$

where we suppress the unnecessary indices. With Eqs. (5), (9), and (10) we can rewrite Eq. (13) as

$$a^2b^2c^2\kappa^2 - d^3L\kappa - d^4 = 0. \tag{14}$$

Separating the roots appearing in Eq. (14) and squaring it once again, we obtain an implicit equation, *i.e.*, a polynomial equation, of a surface that intersects $\mathscr{S}$ with equation (1) along the curves of constant principal curvature. We find the implicit equation of an algebraic surface of degree with two disconnected components:

$$(a^2b^2c^2\kappa^2 - d^4)^2 - d^6\kappa^2L^2 = 0.$$

Thus, we have:

Theorem 4.1. *The curves of constant principal curvature on a one-sheeted hyperboloid are algebraic curves of degree 16. The principal views of these curves are algebraic curves of degree eight.*

Remark: The reduction of the degree of the image curves is caused by the symmetry of the surface $\mathscr{S}$ with respect to the image planes of the three orthogonal projections.

Figure 6 shows the right side view and the front view of the two families of curves of constant principal curvature on the one-sheeted hyperboloid with $a = 1$, $b = \sqrt{2}$, and $c = \sqrt{3}$.

The implicit equation of the curves of constant principal curvature may not be useful for drawing or plotting. Thus, we derive a

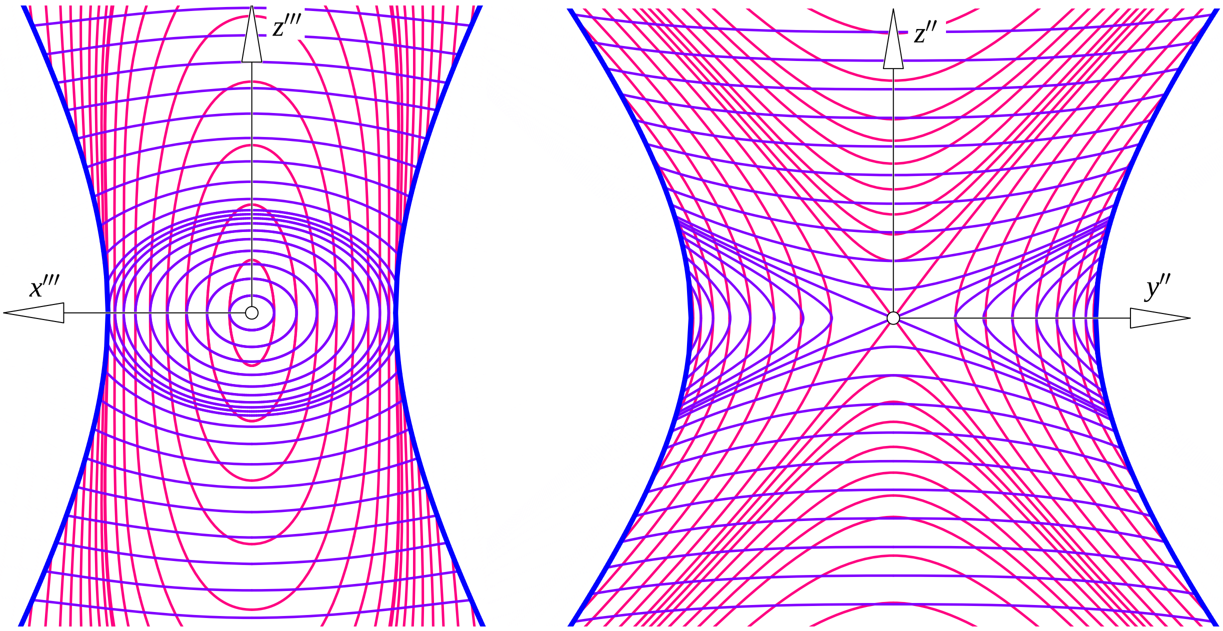

Figure 6: Curves of constant principal curvature (right side view and front view).

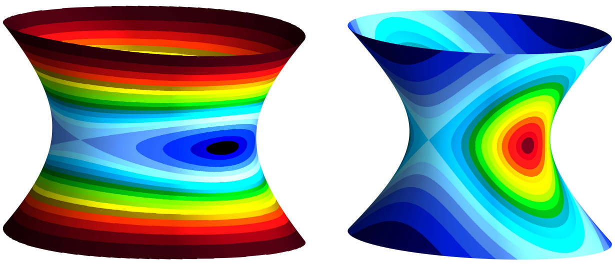

Figure 7: Distribution of principal curvatures on a one-sheeted hyperboloid.

parametrization of these curves. Actually, this parametrization is algebraic (but free of elliptic functions). The iso-curves of the principal curvature are curves on $\mathscr{S}$ (with equation (1)). We parametrize these curves by d, *i.e.*, the support function of the hyperboloid $\mathscr{S}$. Thus, d and the coordinates x, y, z of a point on such a curve are also subject to Eq. (4). Further, the coordinates x, y, and z of a point on an iso-curve of κ fulfill Eq. (10). This system of three quadratic equations is linear in the squares of x, y, and z. The solution of this system of linear equations reads

$$
\begin{aligned}
x^2 &= \frac{a^4}{\beta\gamma\kappa d^3}(d^3 - b^2c^2\kappa)(a^2\kappa + d),\\
y^2 &= -\frac{b^4}{\alpha\gamma\kappa d^3}(d^3 - a^2c^2\kappa)(b^2\kappa + d),\\
z^2 &= \frac{c^4}{\alpha\beta\kappa d^3}(d^3 + a^2b^2\kappa)(d - c^2\kappa).
\end{aligned}
\tag{15}
$$

Here, we have used the fact that Eq. (14) is linear in L, and thus, we have

$$
L = \frac{\kappa^2 a^2 b^2 c^2 - d^4}{\kappa d^3}. \tag{16}
$$

Note that the curves of constant principal curvature are different from the (principal) curvature lines. The latter are characterized by the fact that their tangents are always principal tangents. Furthermore, the two one-parameter families of curvature lines are quartic curves and appear as the intersection of the given quadric with the two one-parameter families of its confocal quadrics as illustrated in Figure 8.

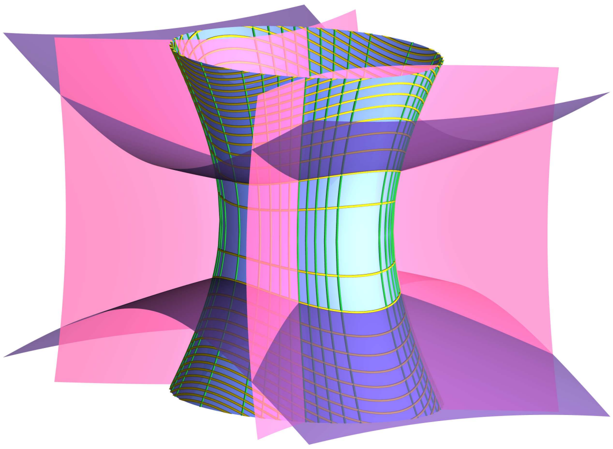

Figure 8: The principal curvature lines do not agree with the curves of constant principal curvature. Principal curvature lines from either family as the intersections of the hyperboloid (light blue) with two confocal quadrics.

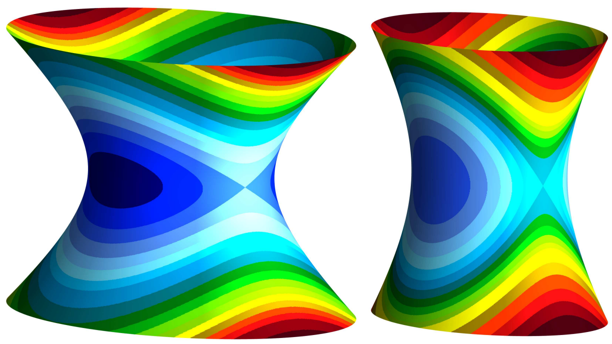

Figure 9: Curves of constant ratio of the principal curvatures on two hyperboloids: $a = 1$, $b = \sqrt{2}$, $c = \sqrt{3}$ (left), $a = c = 1$, $b = \sqrt{2}$ (right).

5. RATIO OF PRINCIPAL CURVATURES

The ratio $R = \frac{\kappa_1}{\kappa_2}$ helps to classify the Dupin indicatrix. If $R = 1$ at some point P on a surface (which will not happen on the one-sheeted hyperboloid), then P is an umbilic and the indicatrix at P is a circle. Since $K = \kappa_1\kappa_2 < 0$ at all points on $\mathscr{S}$, we can only expect $R = -1$. In this case, we have $\kappa_1 = -\kappa_2$, and thus, $M = 0$. At such a point the surface behaves (locally) like minimal surface and the indicatrix consist of a pair of conjugate equilateral hyperbolae.

Figure 9 shows the distribution of the ratio $R = \kappa_1\kappa_2^{-1}$ on two different one-sheeted hyperboloids.

We aim at a precise analytic description of the curves of constant ratio of principal curvature. For that, we solve Eq. (14) for κ and find

$$\kappa_{1,2} = \frac{d^2}{2a^2b^2c^2}\,(dL \mp W)$$

with $W := \sqrt{d^2L^2 + 4a^2b^2c^2}$. Now we let $R = \kappa_1\kappa_2^{-1}$ and derive an implicit equation for the iso-surfaces of R by solving

$$R = (dL - W) : (dL + W)$$

for W and squaring once. Finally, this yields an implicit equation of degree 4 in x, y, and z:

$$a^2b^2c^2(1+R)^2 + Rd^2L^2 = 0. \qquad (17)$$

Now we have:

Theorem 5.1. *The curves of constant ratio of the principal curvatures on a one-sheeted hyperboloid are algebraic curves of degree* 8. *The principal views of the curves of constant ratio of principal curvatures are algebraic curves of degree* 4 *due to the symmetry of* $\mathscr{S}$ *with respect to the principal planes.*

Figure 10 shows the right side view, the front view, and the top view of the iso-curves of the ratio R.

6. CONCLUSION

We have computed the iso-curves of several curvature functions on a one-sheeted hyperboloid. The case of a one-sheeted hyperboloid of revolution is trivial, for the iso-curves of all the functions discussed here are parallel circles.

The iso-curves of the Gauss curvature, the Mean curvature, the two principal curvatures, and the ratio of principal curvatures are algebraic curves on the hyperboloid, indeed on any algebraic surface. This is also the case for the three principal views (orthogonal projections onto a triplet of three mutually orthogonal planes, *i.e.*, in this case the three planes of symmetry). The

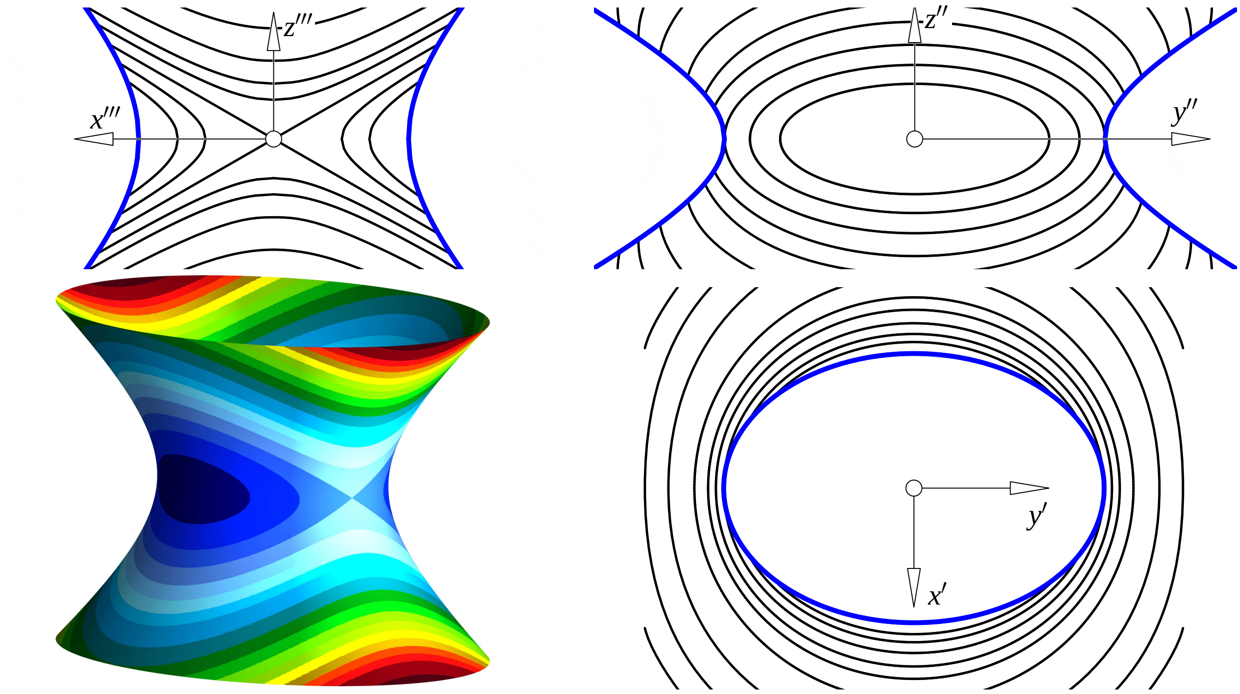

Figure 10: Iso-curves of $\kappa_1 : \kappa_2$ on a hyperboloid ($a = c = 1$, $b = \sqrt{2}$): right side and front view (top row), top view (bottom row, left).

degree of the curves showing up in the principal views are half the degrees of the space curves, since each fibre of any principal projection meets the curve twice.

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ABOUT THE AUTHOR

Boris Odehnal studied Mathematics and Descriptive Geometry at the Vienna University of Technology where he also recieved his PhD and his habilitation in Geometry. After a one-year period as a full interim professor for Geometry at the TU Dresden he changed to the University of Applied Arts Vienna. He can be reached via email at

`boris.odehnal@uni-ak.ac.at`

or at his postal address

Abteilung für Geometrie, Universität für Angewandte Kunst Wien, Oskar-Kokoschka-Platz 2, 1010 Wien, Austria.