3D Shape Understanding and Reconstruction based on Line Element Geometry

B. Odehnal

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- new theory dealing with line elements

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  Euclidean kinematic surfaces (helical (rotational) surf., cylinders),
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- new theory dealing with line elements
- much wider class of surfaces can be detected:
  Euclidean kinematic surfaces (helical (rotational) surf., cylinders),
  equiform kinematic surfaces (spiral surf., general, cones, etc.)
- applicable to recognition/reconstruction of surfaces, segmentation

Related Work.

[1] B. ODEHNAL, H. POTTMANN, J. WALLNER: Equiform kinematics and the geometry of line elements. Submitted to: Contributions to Algebra and Geometry.

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[3] H. POTTMANN, M. HOFER, B. ODEHNAL, J. WALLNER: Line geometry for 3D shape understanding and reconstruction. Proc. ECCV’04.

Equiform Kinematics.

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- equiform motion

  $x \mapsto y = \alpha A x + a \quad \alpha > 0,\ A \in \mathbf{SO}_3,\ a \in \mathbb{R}^3$

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  $\alpha_t : \ I \subset \mathbb{R} \to \mathbb{R},\ A_t : \ I \to \mathbf{SO}_3,\ a_t : \ I \to \mathbb{R}^3$

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  $c \times y + \bar{c} + \gamma y \ldots$ linear in y

  $\dot{A} A^T y = c \times y$, $\gamma = \dot{\alpha}\alpha^{-1}$, $\bar{c} = \dot{a} - \gamma a - c \times a$

Line Elements.

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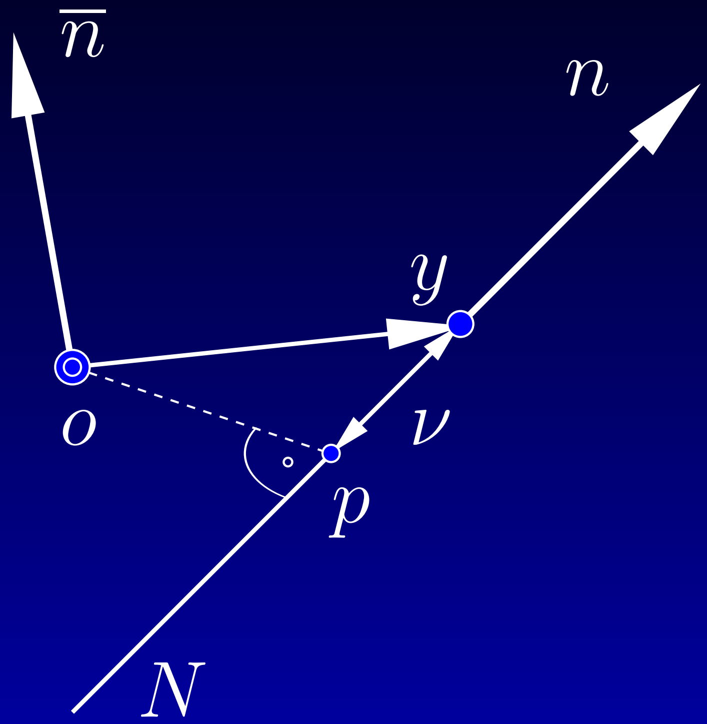

line element (N, y) = line N + point $y \in N$

$(n, \overline{n})$ Plücker coordinates of N, $\overline{n} := y \times n$, $\|n\| = 1$, $\langle n, \overline{n} \rangle = 0$

$\nu = \langle n, y \rangle$
signed distance of p, y

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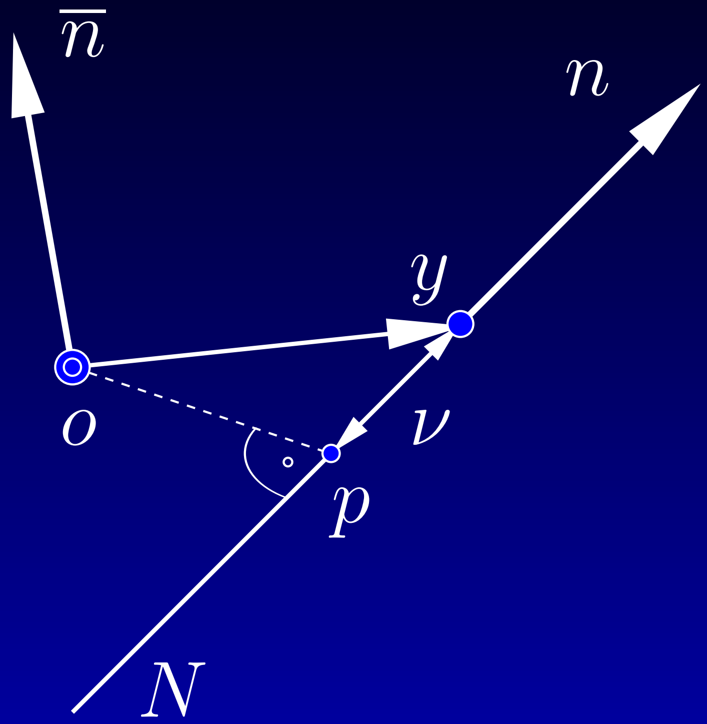

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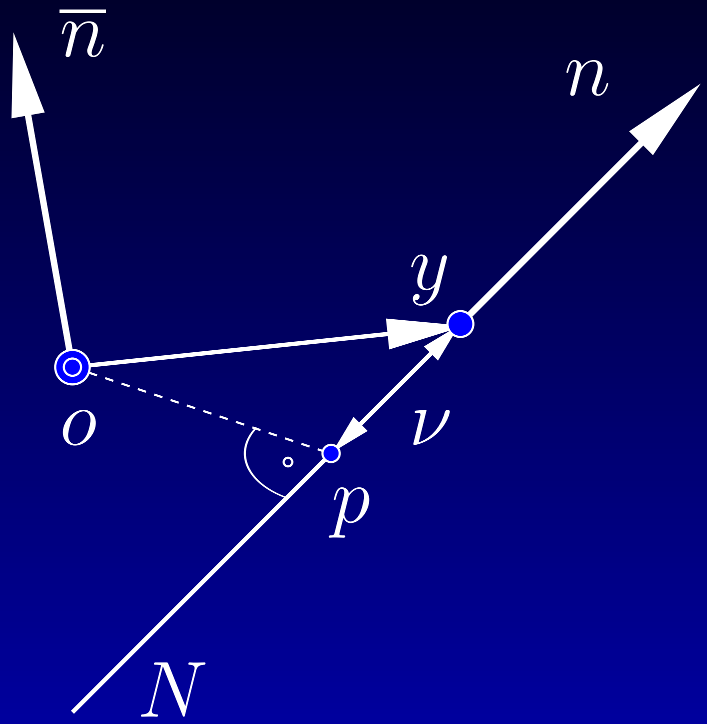

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- $(n, \overline{n}, \nu) \in \mathbb{R}^7$ coordinates of (N, y)
- depend on y, homogeneous $\Longrightarrow$ point model in $\mathbb{P}^6$: quadratic cone (vertex, one generator missing, see [1])

Linear Complexes of Line Elements.

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Definition:
The set C of line elements $(N, y) = (n, \overline{n}, \nu)$ with

$$\langle n, \overline{c} \rangle + \langle \overline{n}, c \rangle + \nu\gamma = 0$$

is called a linear complex of line elements.
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Lemma:
For given $C = (c, \overline{c}, \gamma)$ ($\gamma \neq 0$) and each line $L = (l, \bar{l})$, there is a uniquely defined point x, s.t. $(L, x) = (l, \bar{l}, \lambda) \in C$.

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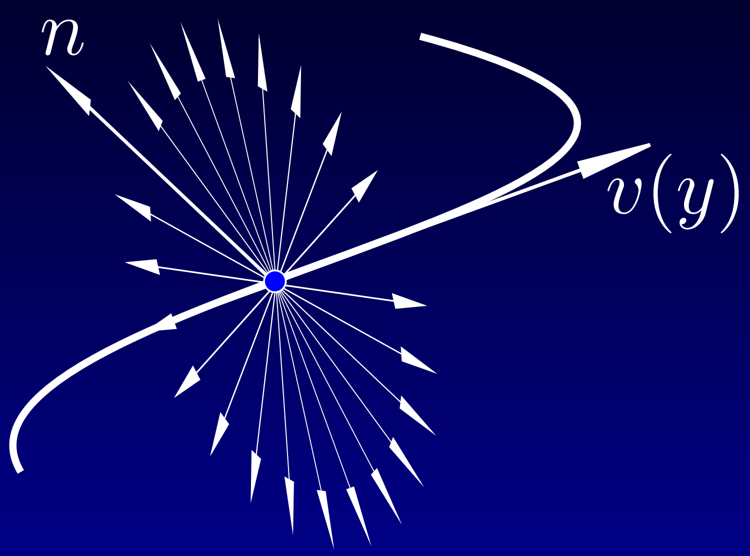
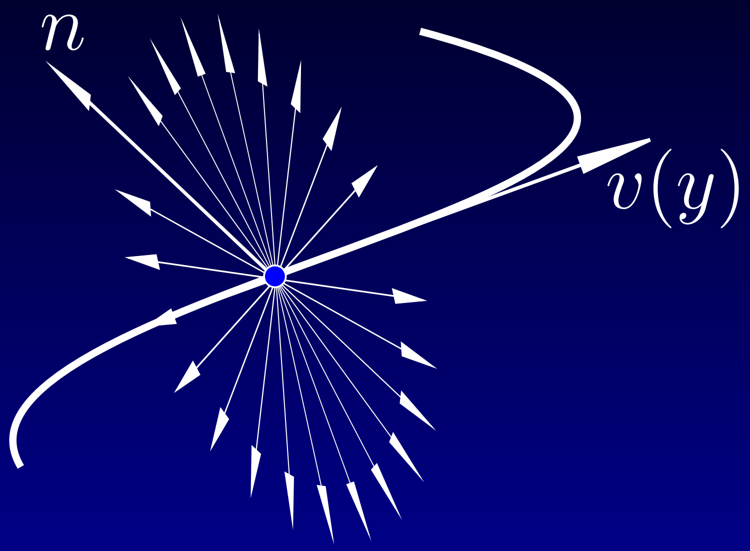
Proof: $\lambda = -(\langle \overline{c}, l\rangle + \langle c, \bar{l}\rangle)/\gamma$. $\square$

Linear Complexes & Equiform Kinematics.



Theorem:
At any regular instant of a smooth 1-param. equiform motion the path normal elements form a linear complex of line elements.

Linear Complexes & Equiform Kinematics.



Theorem:
At any regular instant of a smooth 1-param. equiform motion the path normal elements form a linear complex of line elements.

Proof:
$N(y) \ldots (n, \overline{n} = y \times n)$ path normal at y,
$N(y) \perp v(y) \iff 0 = \langle n, c \times y + \overline{c} + \gamma y \rangle =$
$\langle \overline{n}, c \rangle + \langle n, \overline{c} \rangle + \gamma\nu = 0.$ $\square$

Properties of linear Complexes of Line Elements.

Given a linear complex of line elements $(c, \overline{c}, \gamma)$:

- line elements on lines of a bundle:
  points form a sphere

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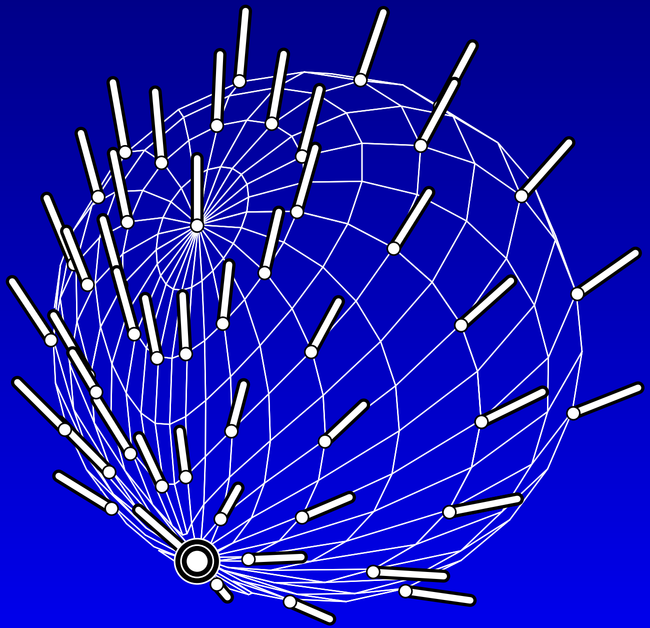

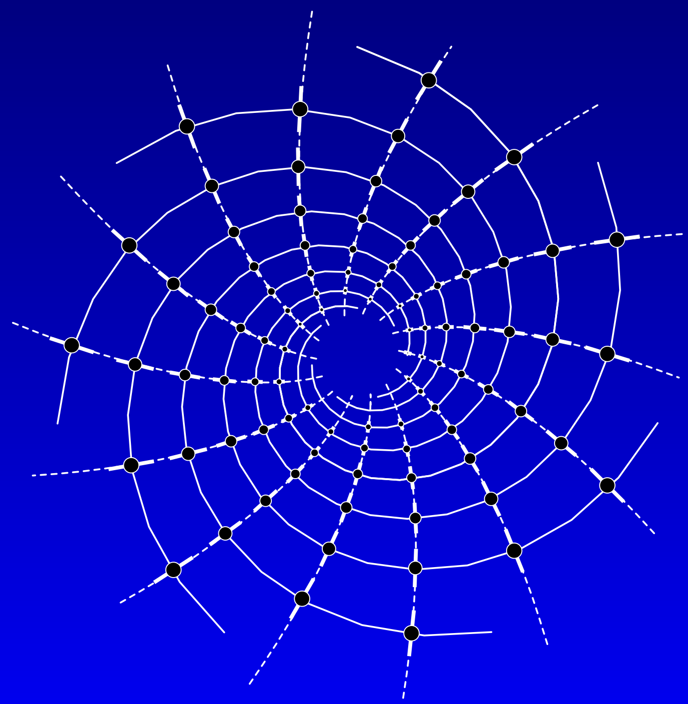

Types of Complexes & corresponding uniform equif. motions.

up to equiform equivalence

- $C = (0, 0, 1; 0, 0, p; 0) \quad p \in \mathbb{R}$
  helical motion ($p \neq 0$), rotation ($p = 0$)

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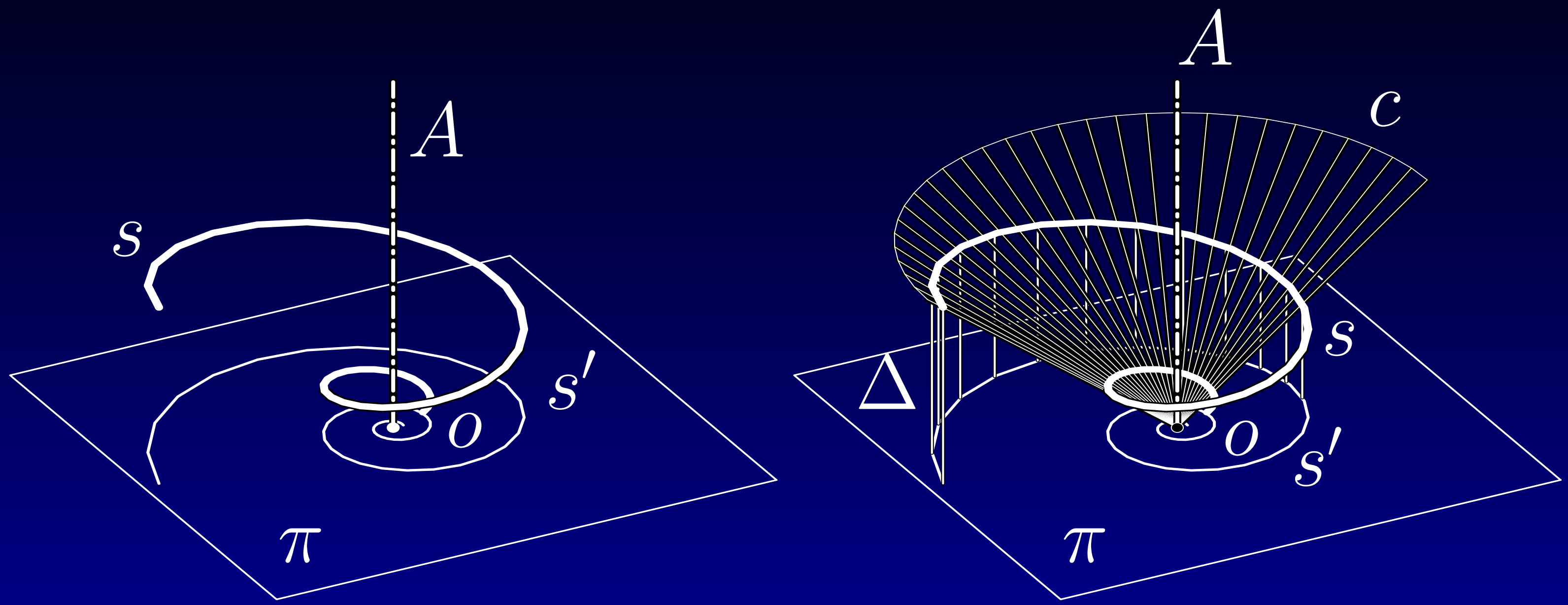
  translation
- $C = (0,0,1;0,0,0;p) \quad p \in \mathbb{R}$
  spiral motion ($p \neq 0$), rotation ($p = 0$)
- $C = (0,0,0;0,0,0;1)$
  similarity transformation

Spiral Motion.



- o = fixed point, A = axis, π = fixed plane

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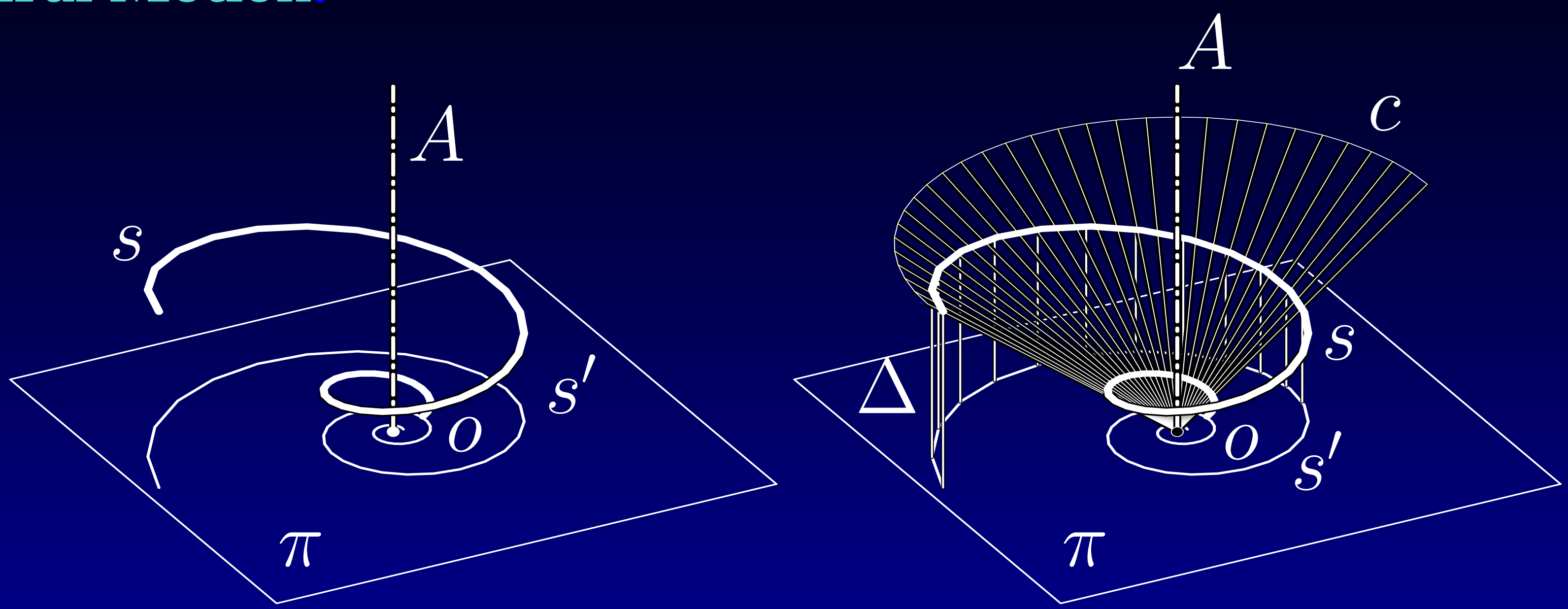

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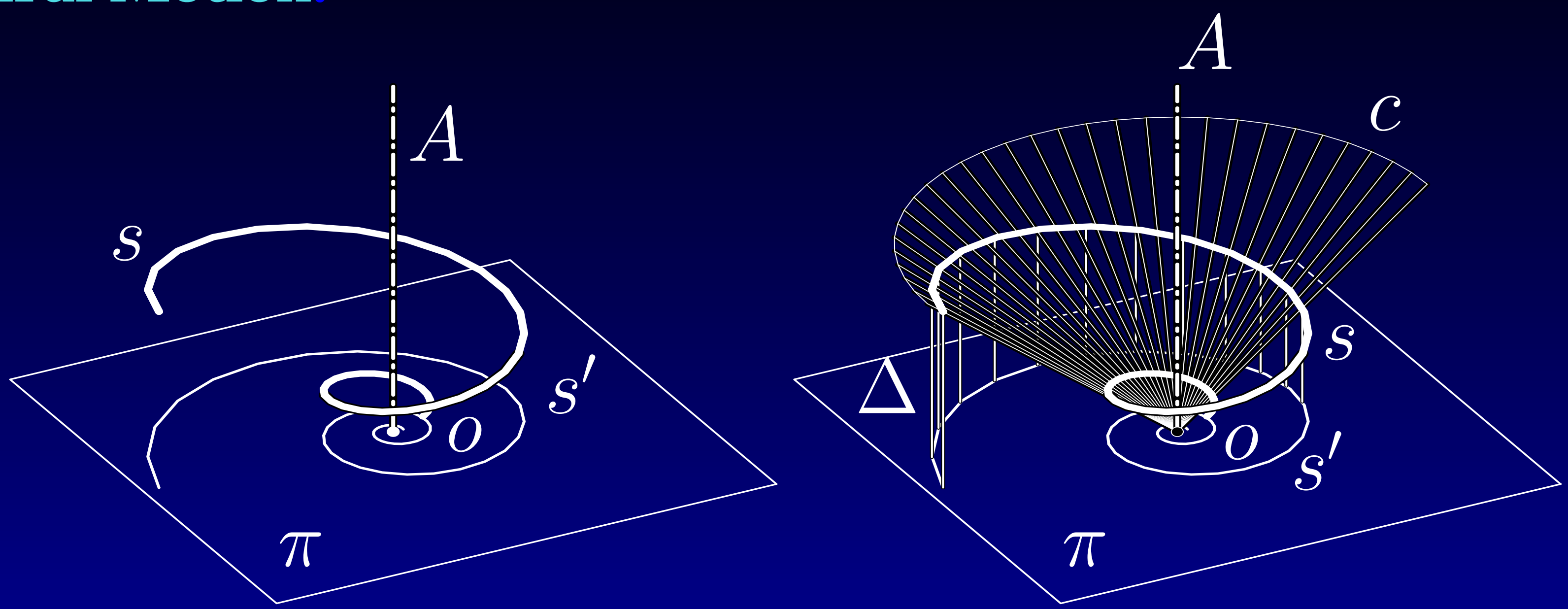

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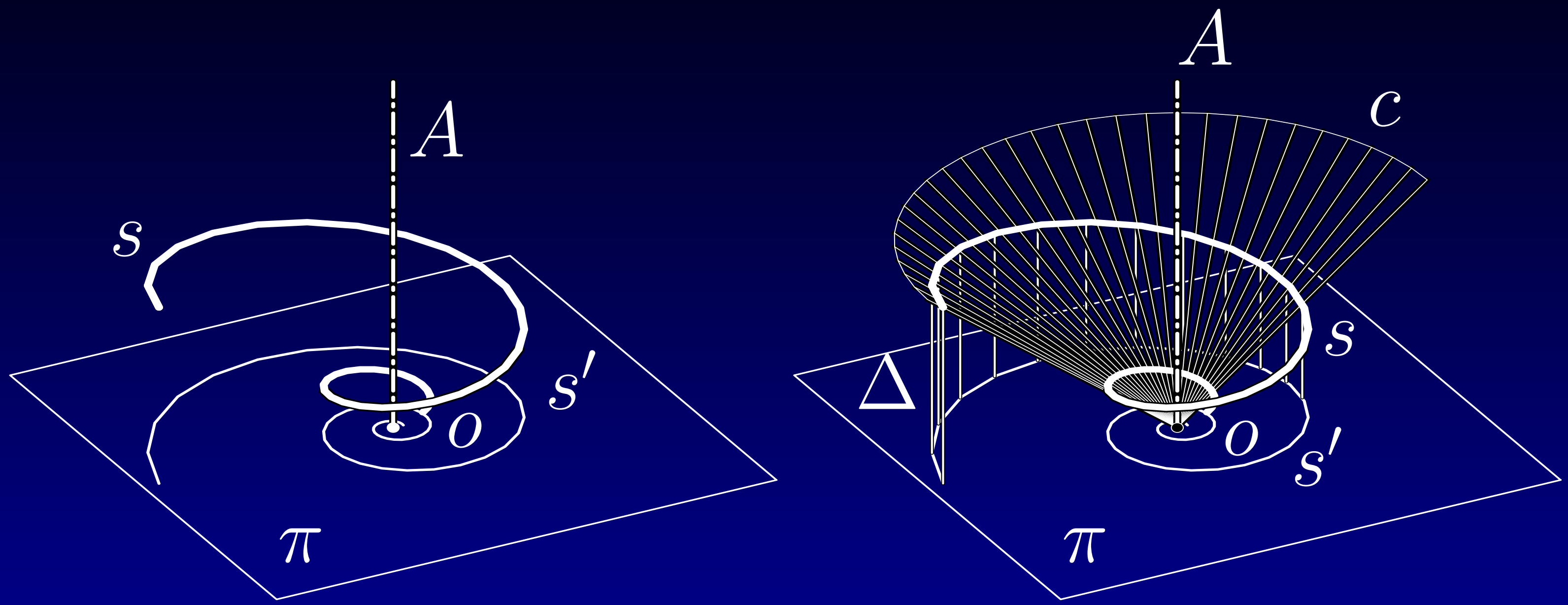

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- Δ = invariant spiral cylinder

Recognition of Surfaces.

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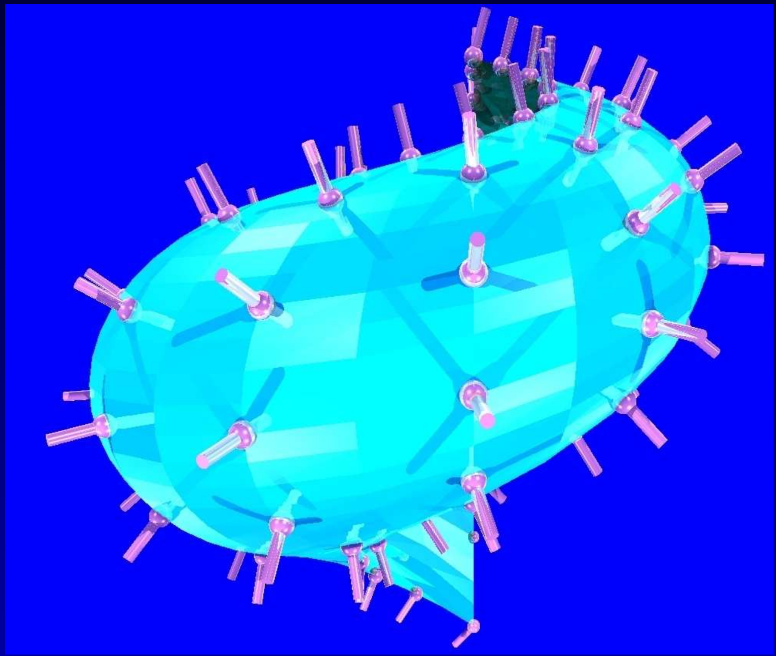

Theorem:
The normal elements of a regular C^1 surface are contained in a linear complex of line elements if and only if it is part of an equiform kinematic surface.

Recognition / exact case.

- given: $(n_i, \overline{n}_i, \nu_i) \ldots$ normal elements of an equiform kinematic surface S

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- compute a basis in the space V of linear equations $\langle c, \overline{x}\rangle + \langle \overline{c}, x\rangle + \xi\gamma = 0$

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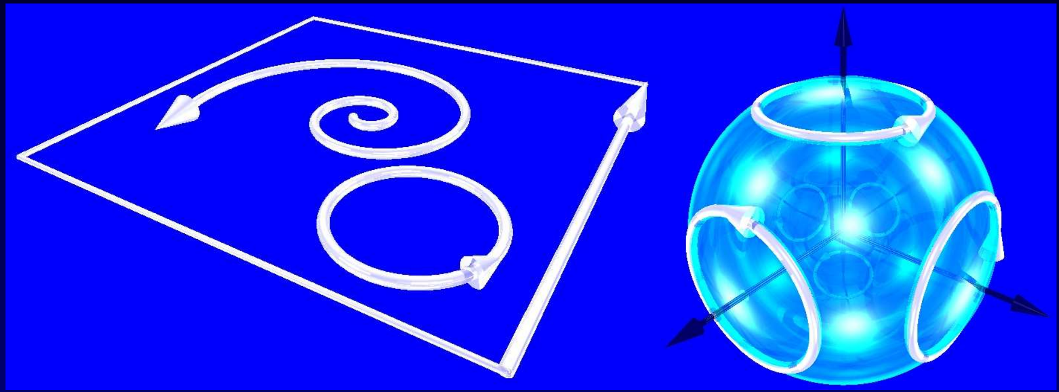
- independent basis vectors = independent unif. equif. motions $S \to S$

Invariant Surfaces.



$k = 4$: plane, $k = 3$: sphere

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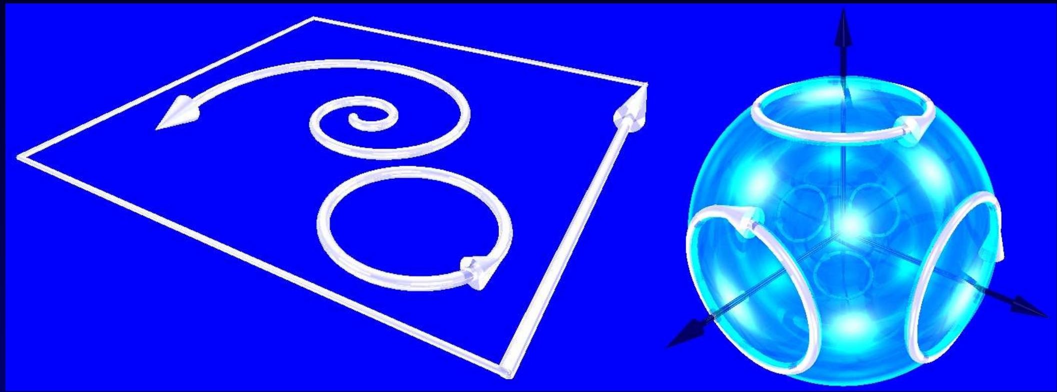

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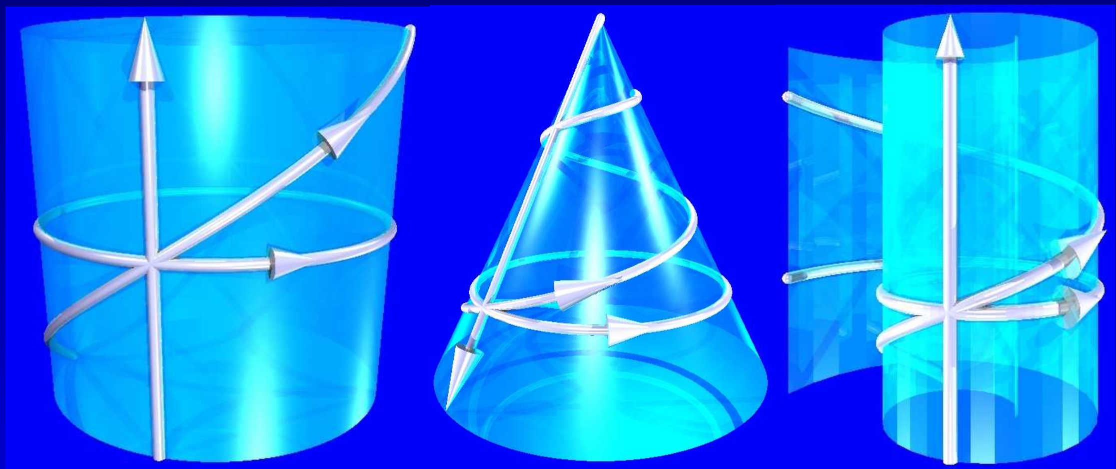
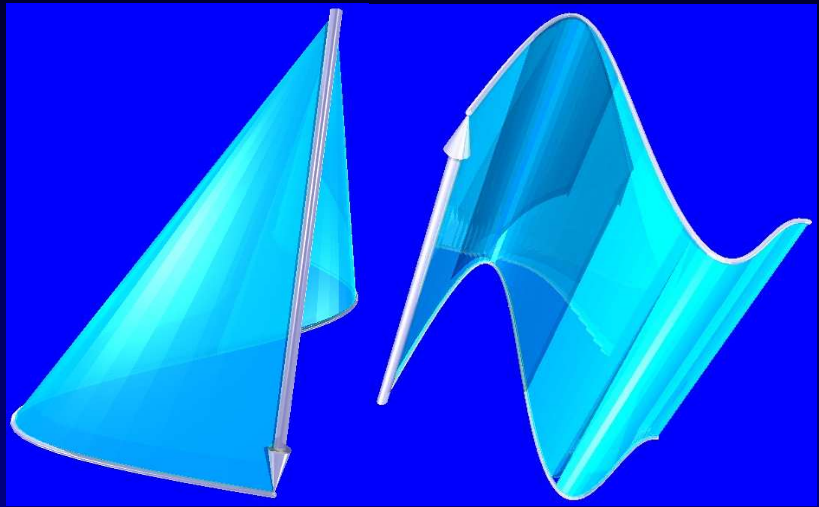


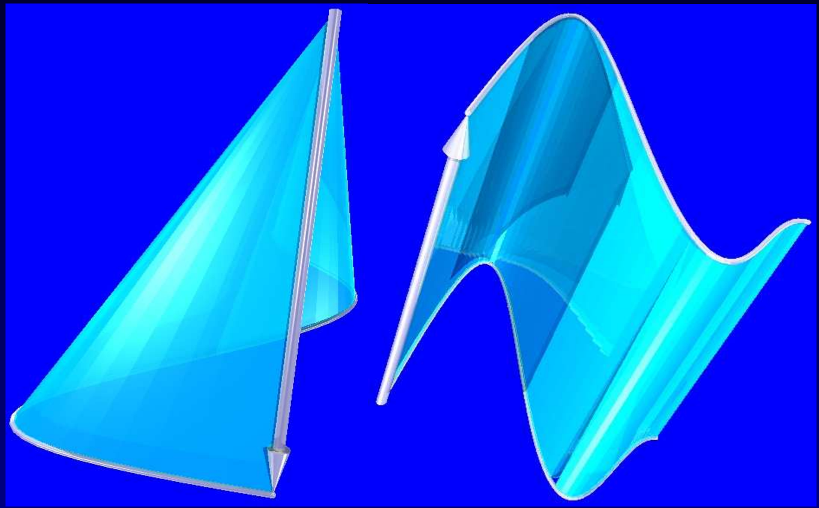
$k = 2$: cylinder/cone of rev., spiral cylinder

Invariant Surfaces.



$k = 1$: general cone/cylinder

Invariant Surfaces.



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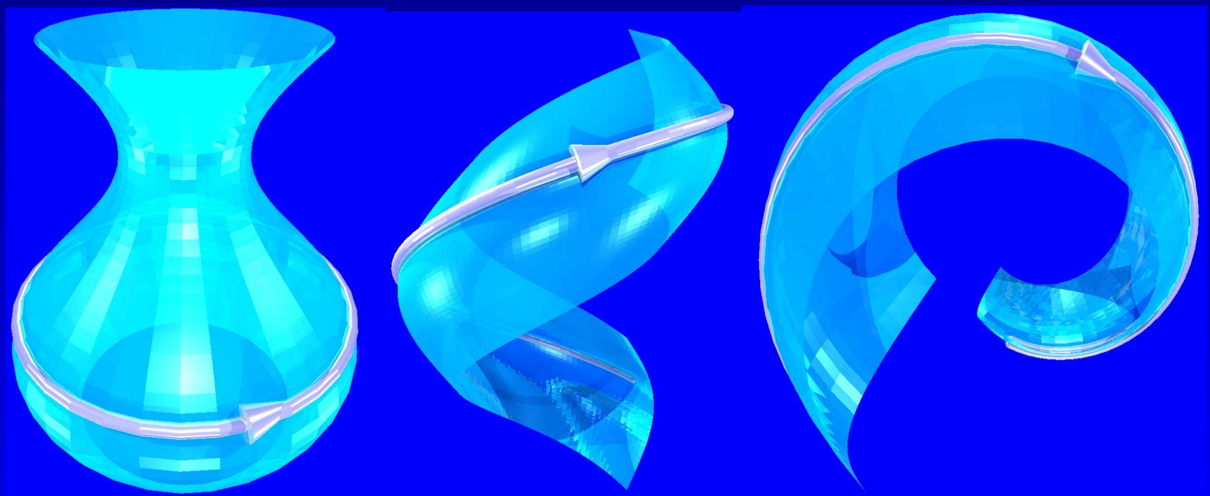
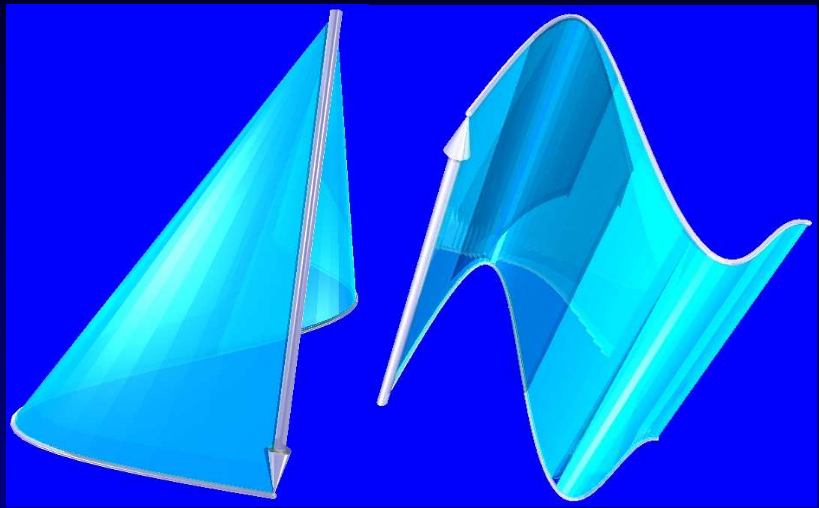


$k = 1$:

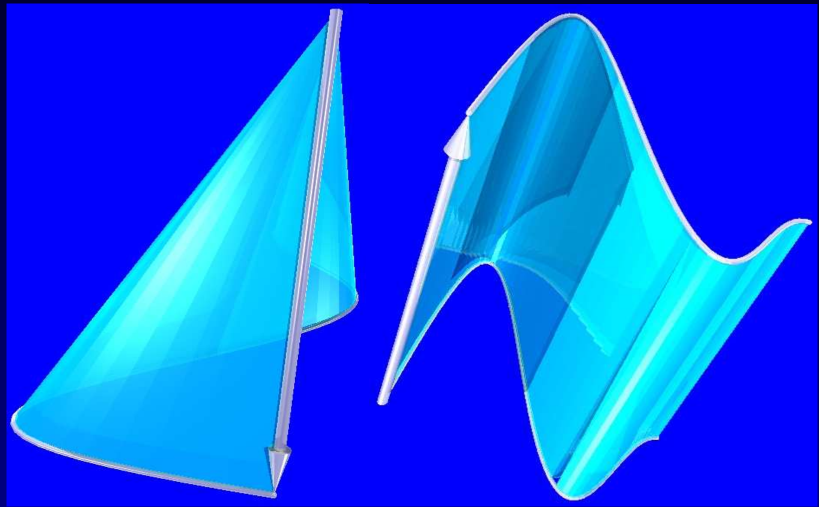
rotational/helical surface, spiral surface

Invariant Surfaces.



$c = 0, \gamma \neq 0 \quad c = 0, \gamma = 0$

Invariant Surfaces.



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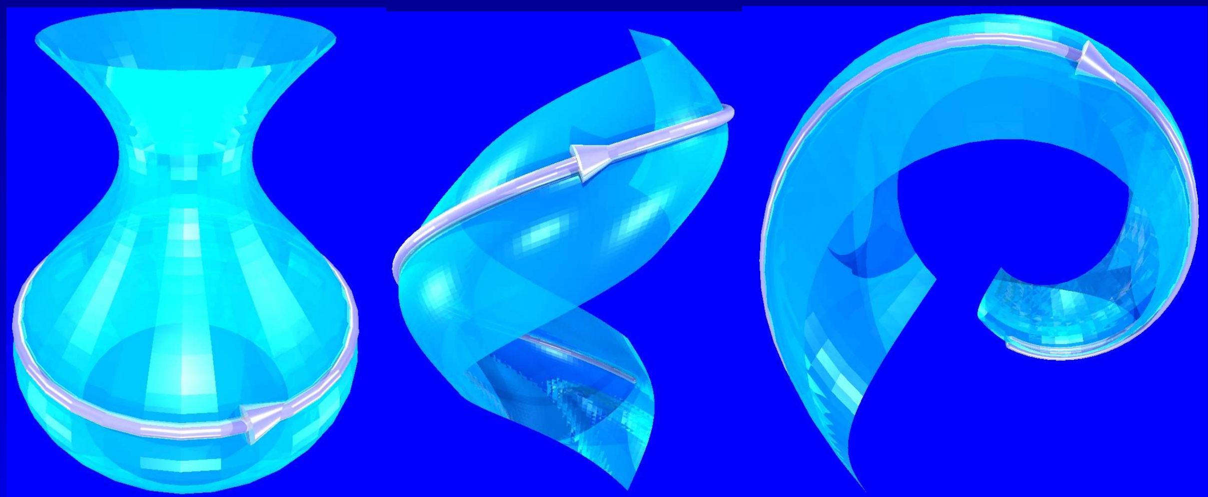

$c \neq 0, \langle c, \overline{c} \rangle = \gamma = 0 \quad c \neq 0, \gamma = 0, \langle c, \overline{c} \rangle \neq 0 \quad c \neq 0, \gamma \neq 0$

Recognition / non-exact case.

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  wanted: best approx. equiform kinematic surface

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- minimizing the sum of squared moments

$$\sum_{i=1}^{N} \mu_i^2 := \sum_{i=1}^{N} (\langle c, \overline{n}_i \rangle + \langle \overline{c}, n_i \rangle + \nu_i \gamma)^2$$

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- number/type of eigenvectors corresponding to small eigenvalues of
$$\sum_{i=1}^{n} (\overline{n}_i, n_i, \nu_i)^T (\overline{n}_i, n_i, \nu_i)$$

determine type of best approx. kinematic surface

Recognition / non-exact case.

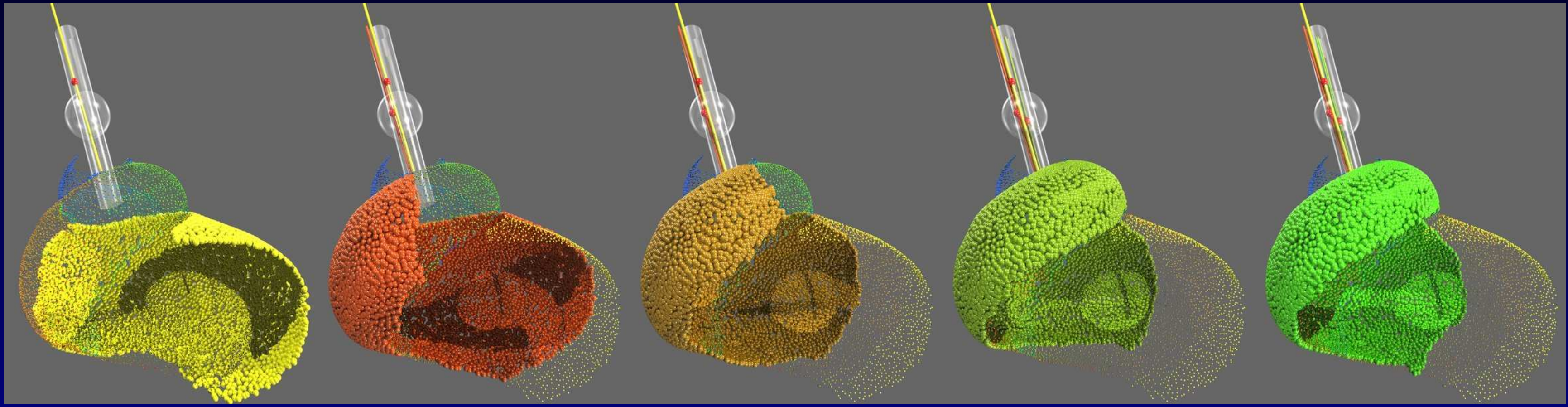
- using RANSAC

Recognition / non-exact case.

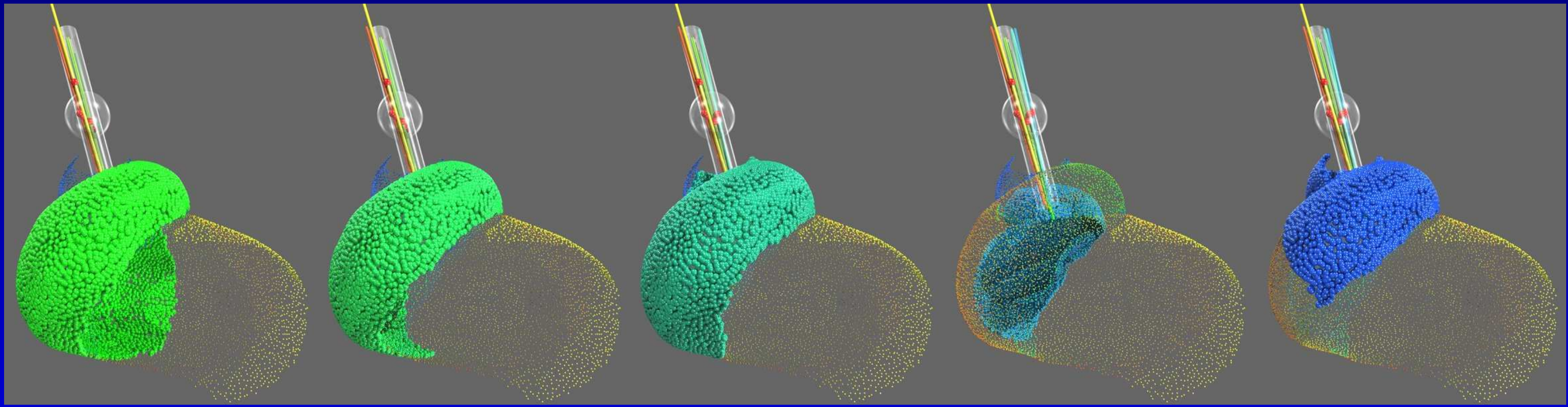
- using RANSAC
- downweighting outliers with

$$w_i = \frac{1}{1 + F\mu_i^{2k}}, \quad F > 0$$

Recognition / non-exact case



color of region = color of axis

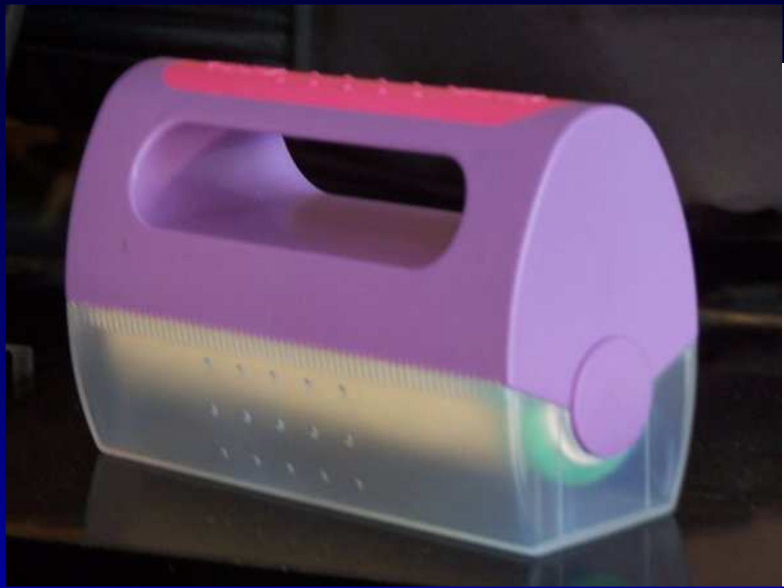
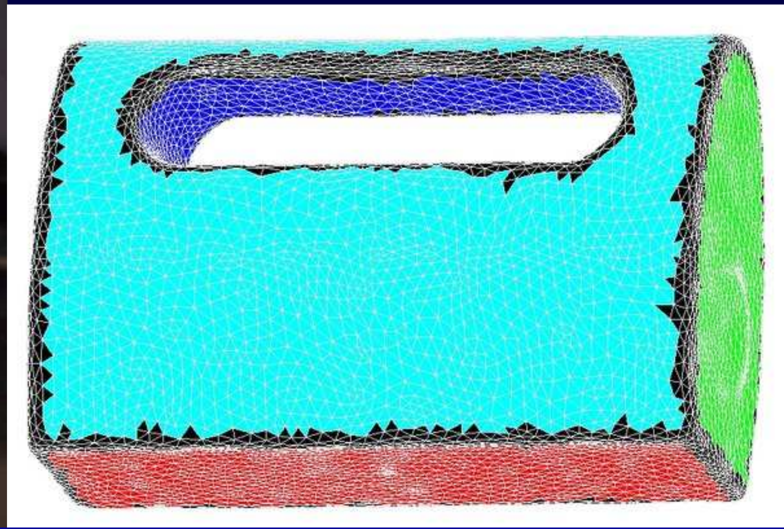


centers and axis within the transparent fat line element

Segmentation.

Segmentation.

- removing sharp edges

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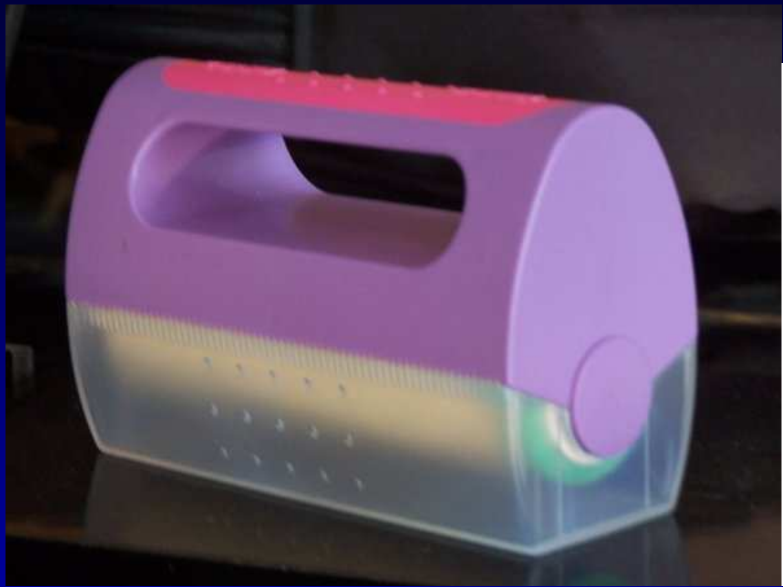
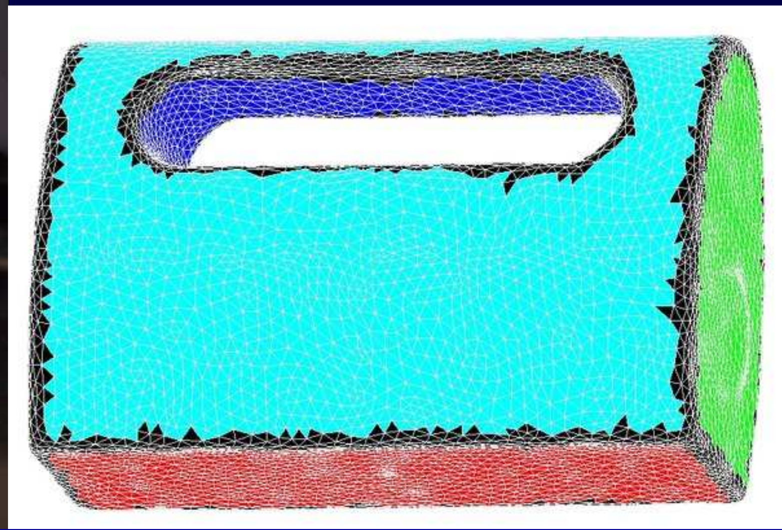

- looking for plane/sphere/cylinder (cone) of rev.

Segmentation.

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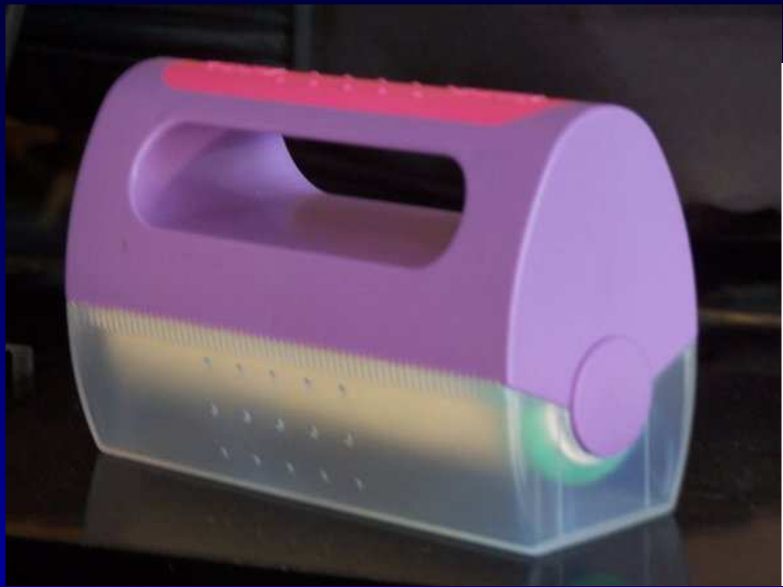 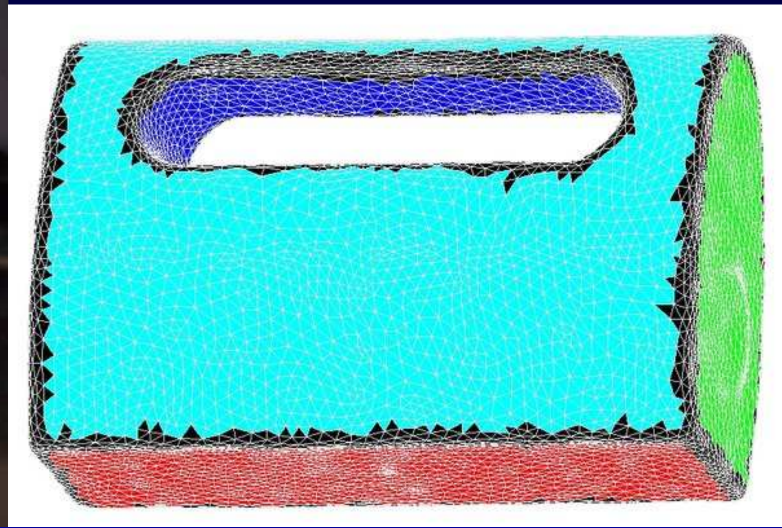

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- looking for multiply invariant surfaces

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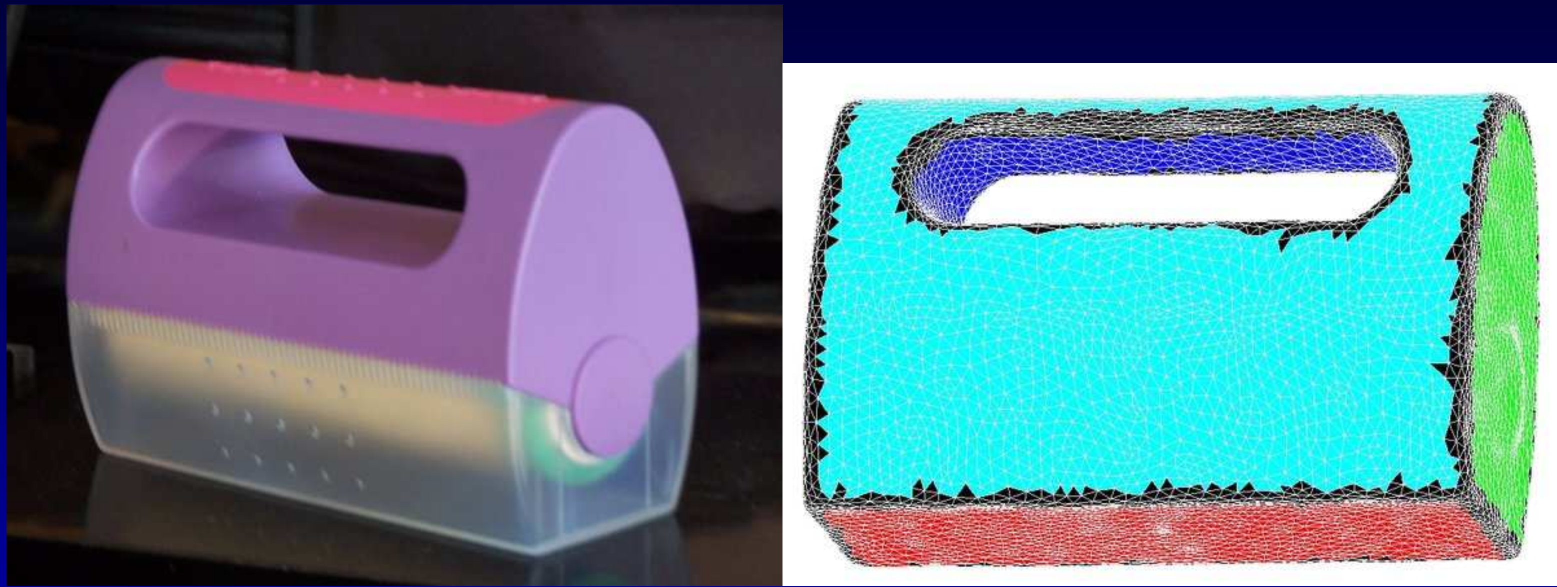

- looking for plane/sphere/cylinder (cone) of rev.
- looking for multiply invariant surfaces
- looking for simply invariant surfaces

Extracting geometric Data.

- axis A and center o of spiral motion:

$$v(z) = 0 = c \times z + \overline{c} + \gamma z$$

$$z = \frac{1}{\gamma(c^2+\gamma^2)}(\gamma c \times \overline{c} - \gamma^2 \overline{c} - (c \cdot \overline{c})c)$$

$$A = (c, \frac{1}{\gamma^2 + c^2}(c^2 \overline{c} - (c \cdot \overline{c})c + \gamma c \times \overline{c}))$$

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- axis A and center o of spiral motion:

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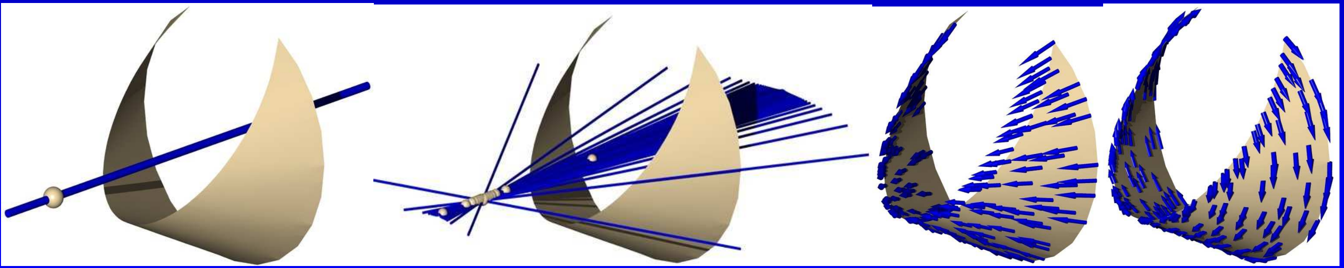
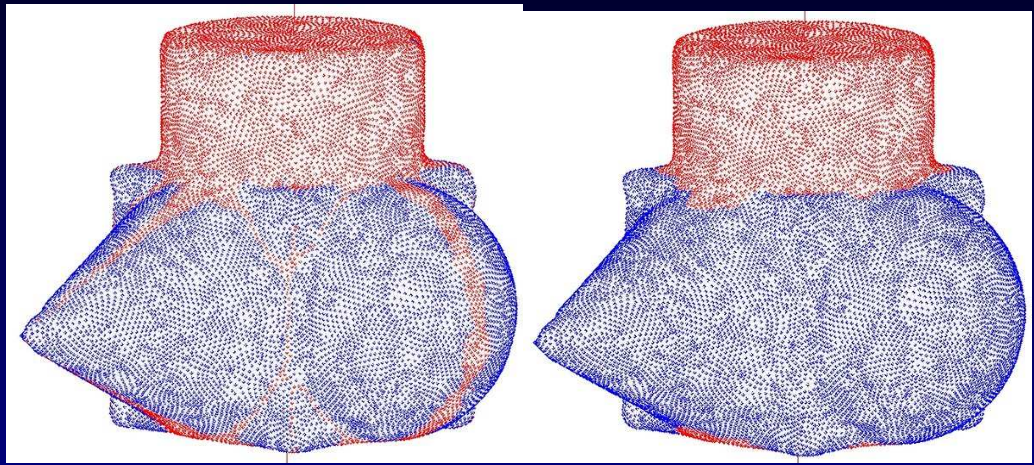
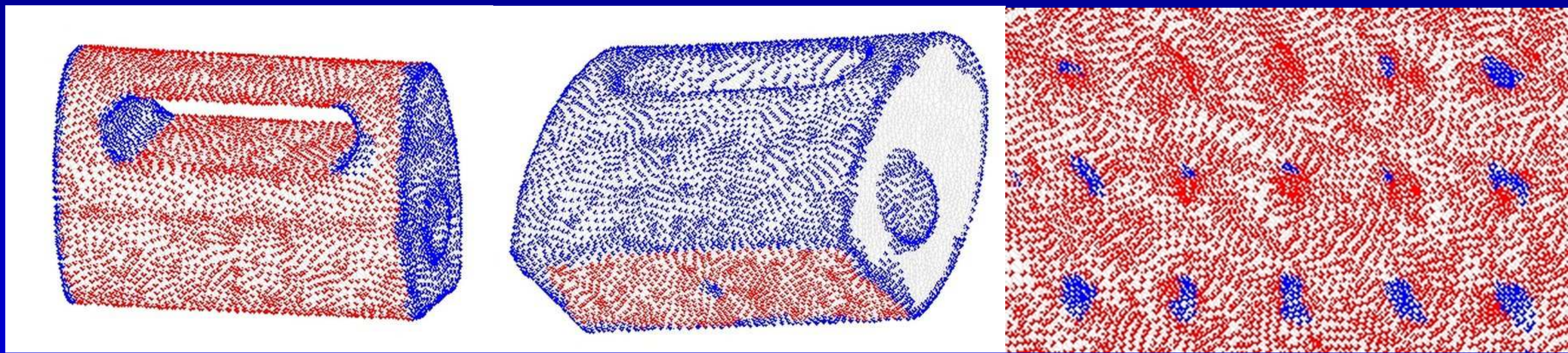


Segmentation / Examples



detecting rotational surface, morphologhical operations



detecting a general cylinder, planar parts, and small features

Reconstruction.

Reconstruction (spiral surfaces).

- $(c, \overline{c}, \gamma) \ldots$ best approx. line element complex

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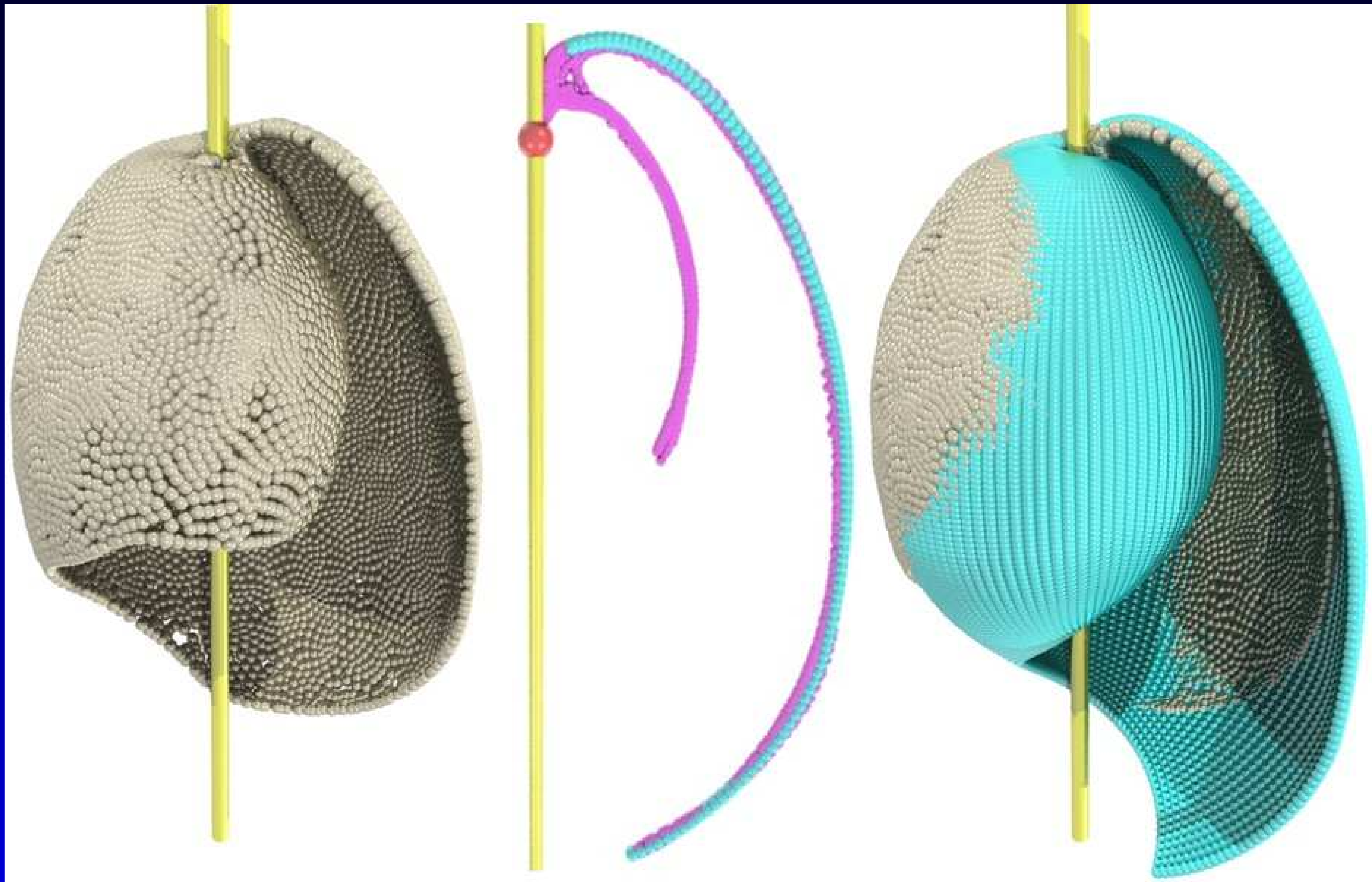
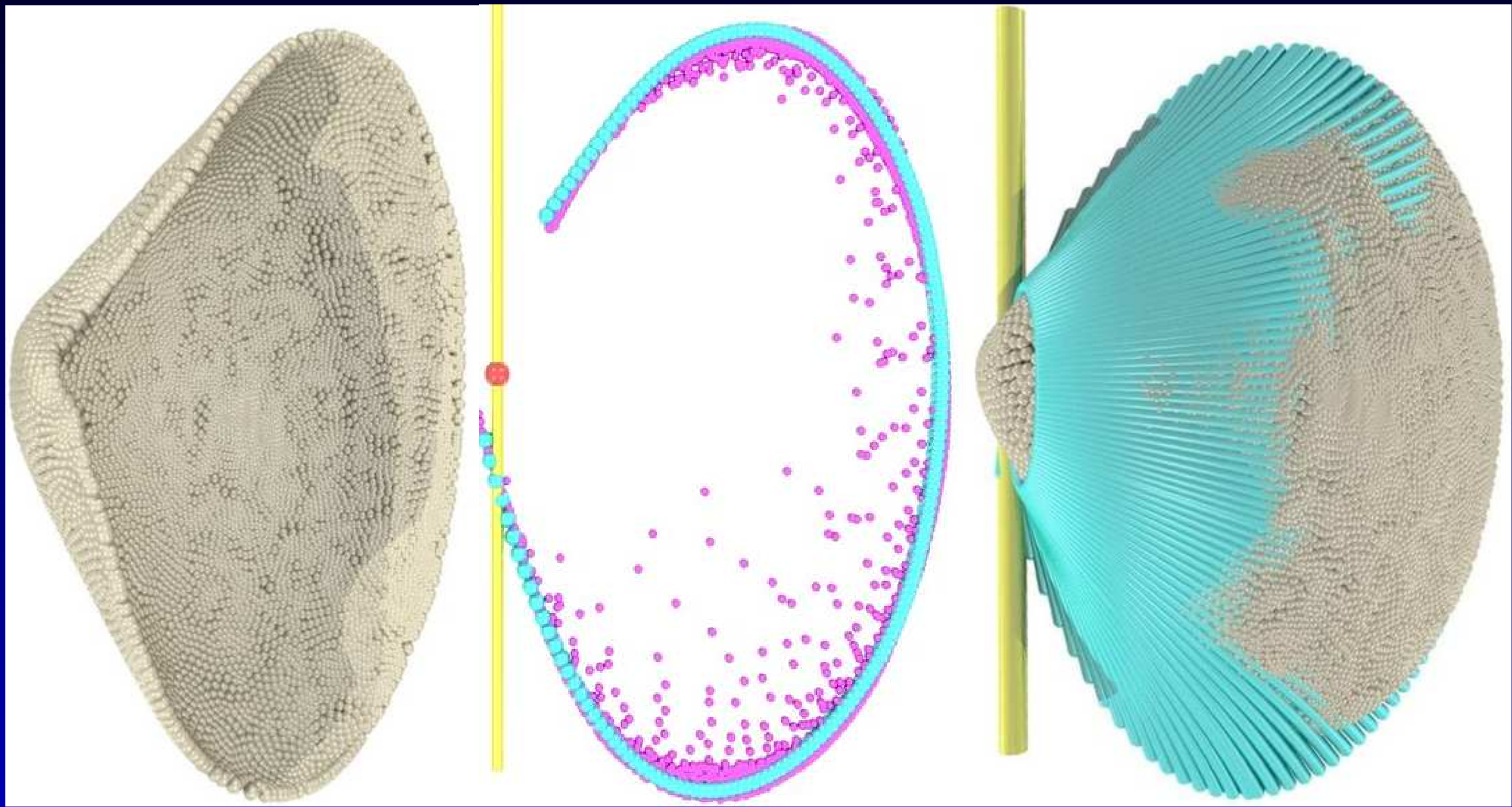
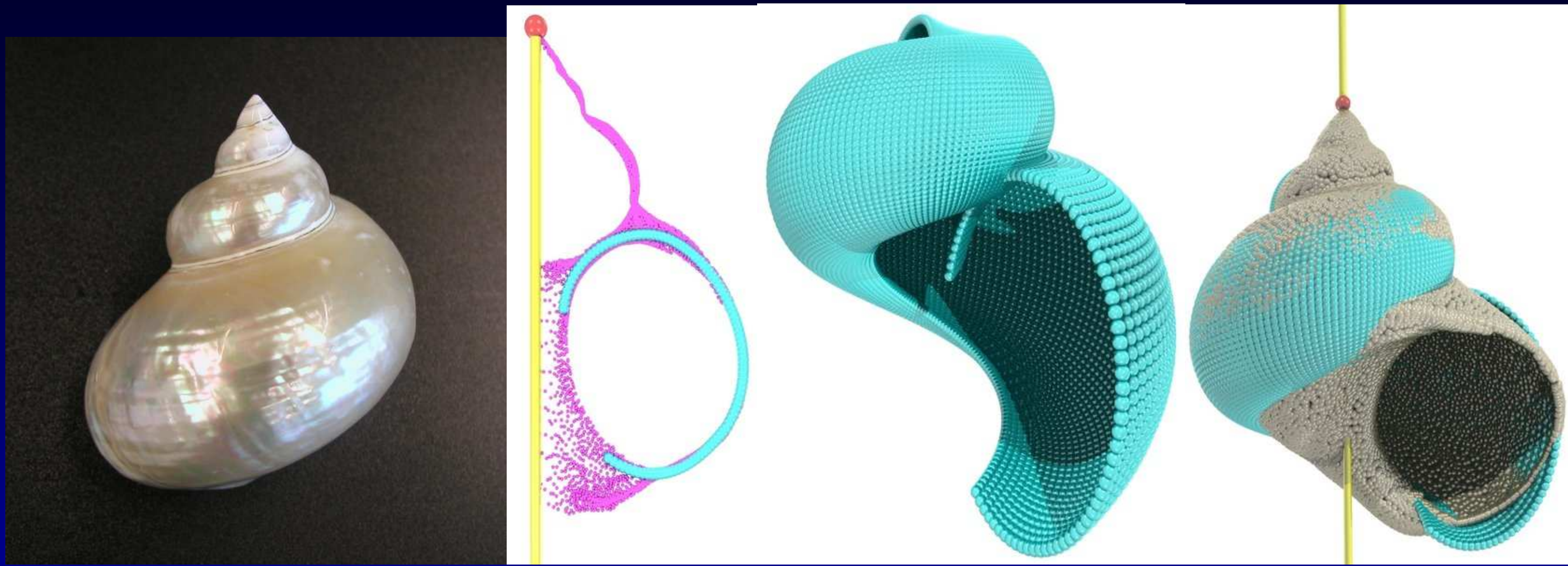


Reconstruction.



Reconstruction



Overview.

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- Aims
- Equiform Kinematics
- Line Elements
- Linear Complexes of Line Elements
- Recognition of Surfaces
- Segmentation
- Reconstruction