Rectification of an Edgy Photograph

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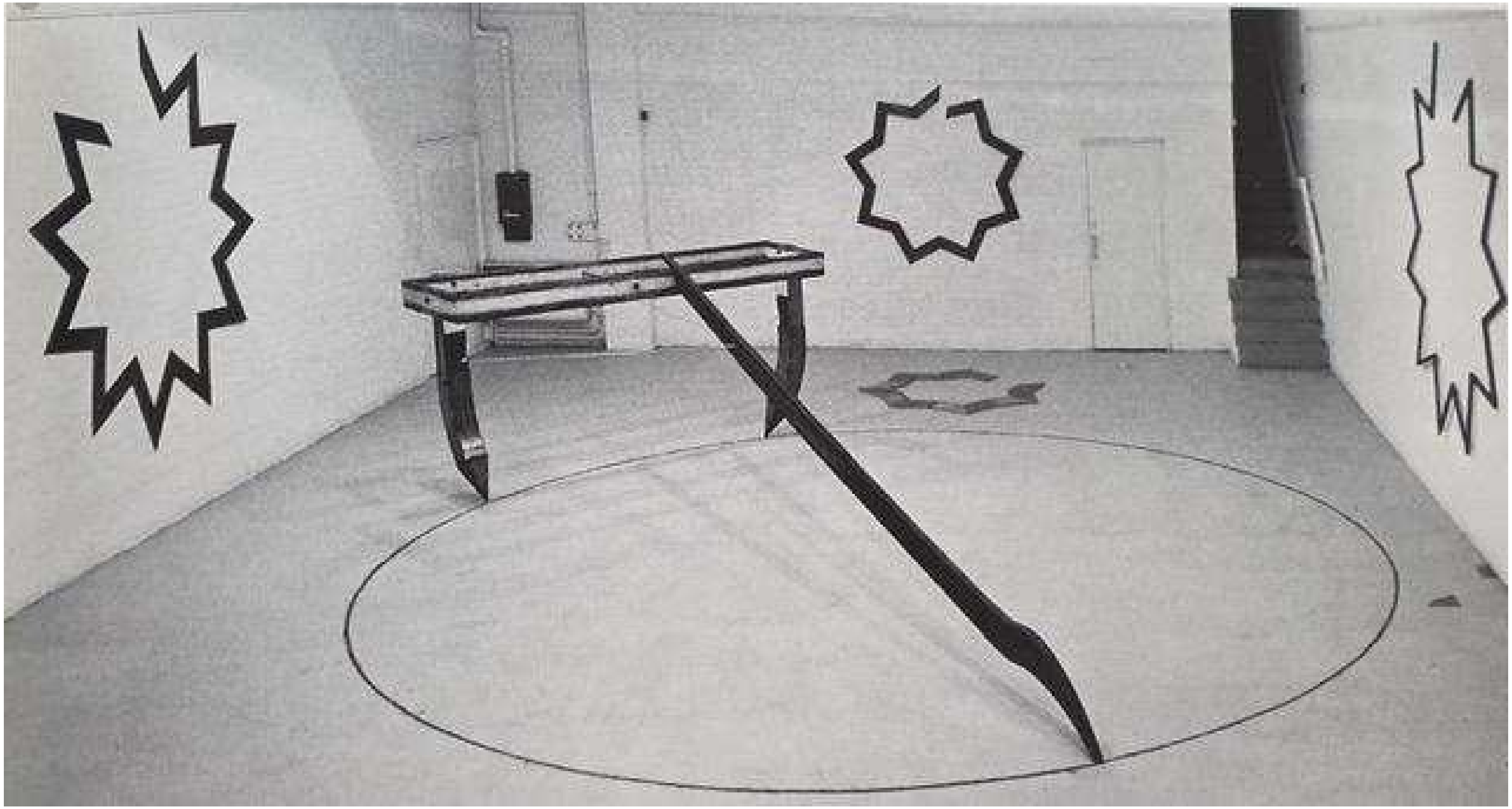
rough sketch of the talk

edgy photograph	motivation
image portions	good and bad choice
basic knowledge	geometric reconstruction
conjugate diameters	rectification of a circle, elliptic involutions
naive approach	and why it failed
geometry succeeds	at last

The edgy photograph



“Ramp and Hyphen” by Paul Neagu. The picture was taken during an exhibition in Glasgow (Scotland) in 1978 (with kind permission of Toni Neagu) [8].

Why do we use/prefer the constructive approach? [4,5,6,11,12,13,14]

simplicity

needs only paper and pencil

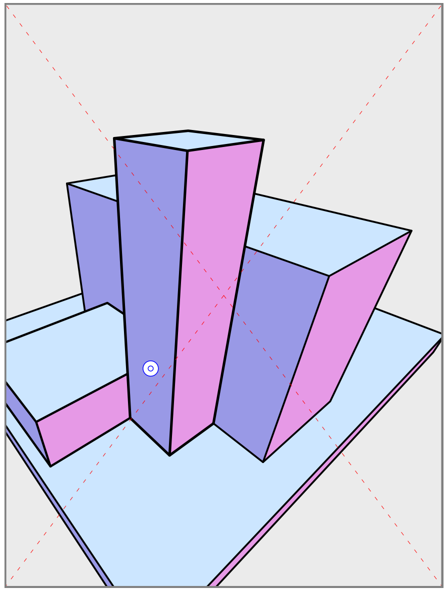
geometric knowledge vs. algorithms

Do algorithms check the plausibilty of results?

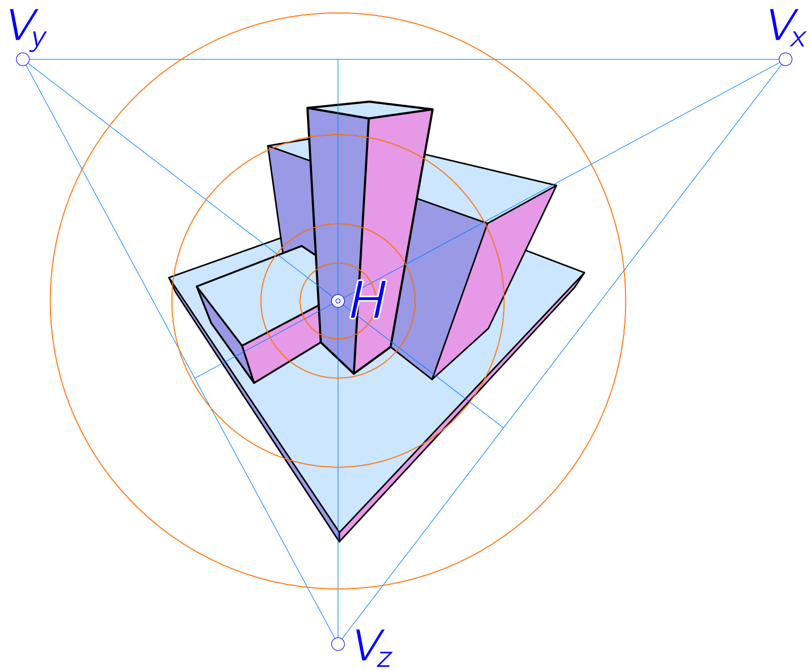
plea for (descriptive) geometry

Geometric knowledge is endangered of getting lost.

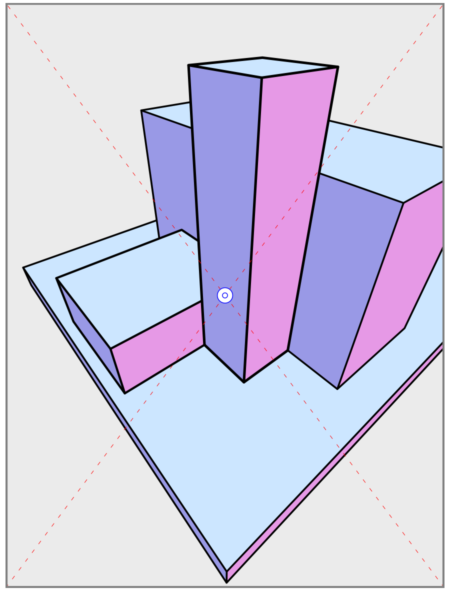
Good and badly chosen portions



improperly chosen portion of the image



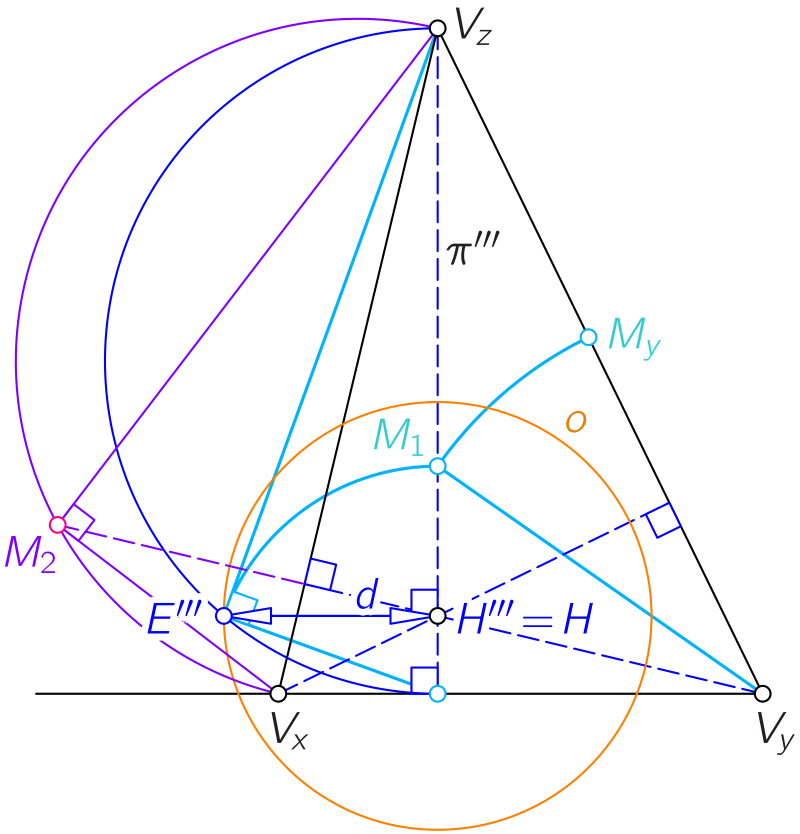
perspective image with circles marking the traces of the viewing cones with semi apertures 45°, 30°, 15°, 7.5°



properly chosen display window

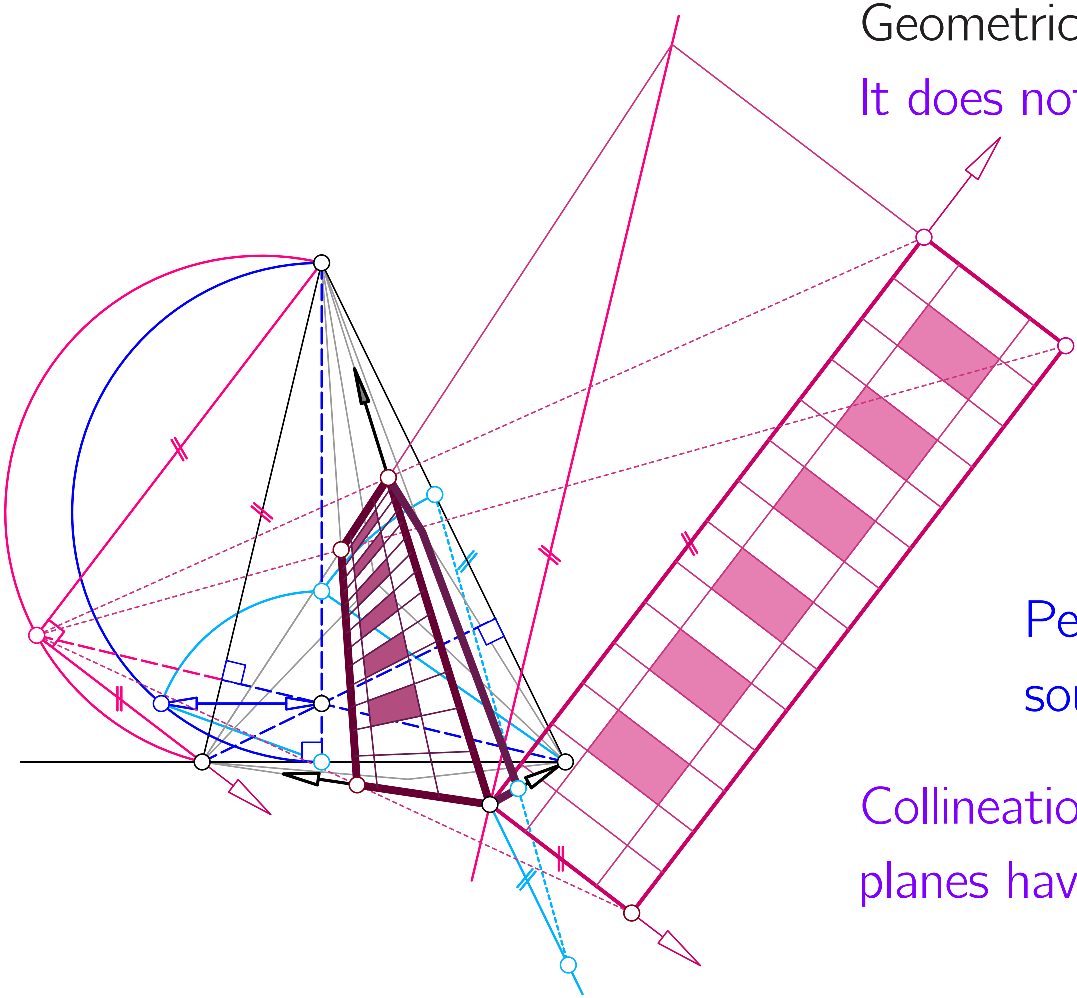
The basic techniques

[1,2,7,9,10,15,16]



1. determine principal vanishing points, i.e., vanishing points V_x, V_y, V_z of a triple of mutually orthogonal lines
2. principle point H = orthocenter of $V_xV_yV_z$
3. (eye) distance d via side view, distance circle o
4. measurement points = centers of perspective collineations that rectify planar figures

This allows for the construction of measurement points for any plane and any line.
For example: M_1 (horizontal planes $\parallel \pi_1 = [x, y]$), M_y (y-direction in $\pi_2 = [y, z]$), ...



Geometric rectification is only up to scale.
It does not fail with perfect images.

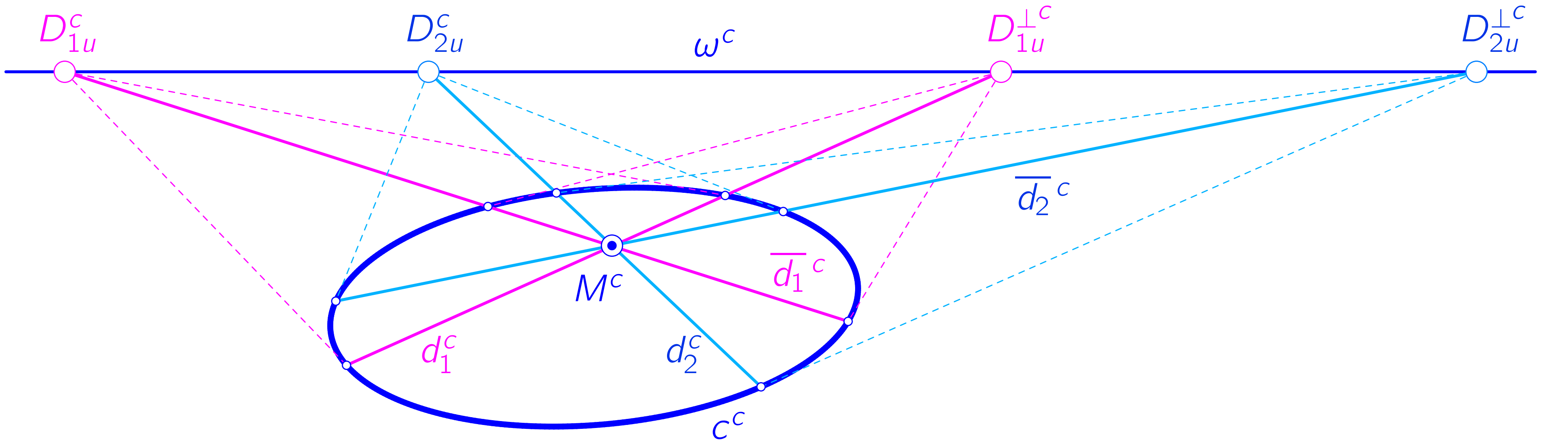
The rectification uses perspective collineations with measurement points for their centers and the vanishing lines (image of a plane's ideal line) as their vanishing lines.

Perspective collineations are sources of inaccuracies.

Collineations that rectify images in different planes have to be made consistent.

Conjugate diameters

[3,10]



The center M of a conic c is the pole of the ideal line ω w.r.t. c.

Each line through M is a diameter of c.

(d_1, d_2) is a pair of conjugate diameters of a conic c if d_1 and d_2 are conjugate w.r.t. c, *i.e.*, d_i contains d_j's pole D_j.

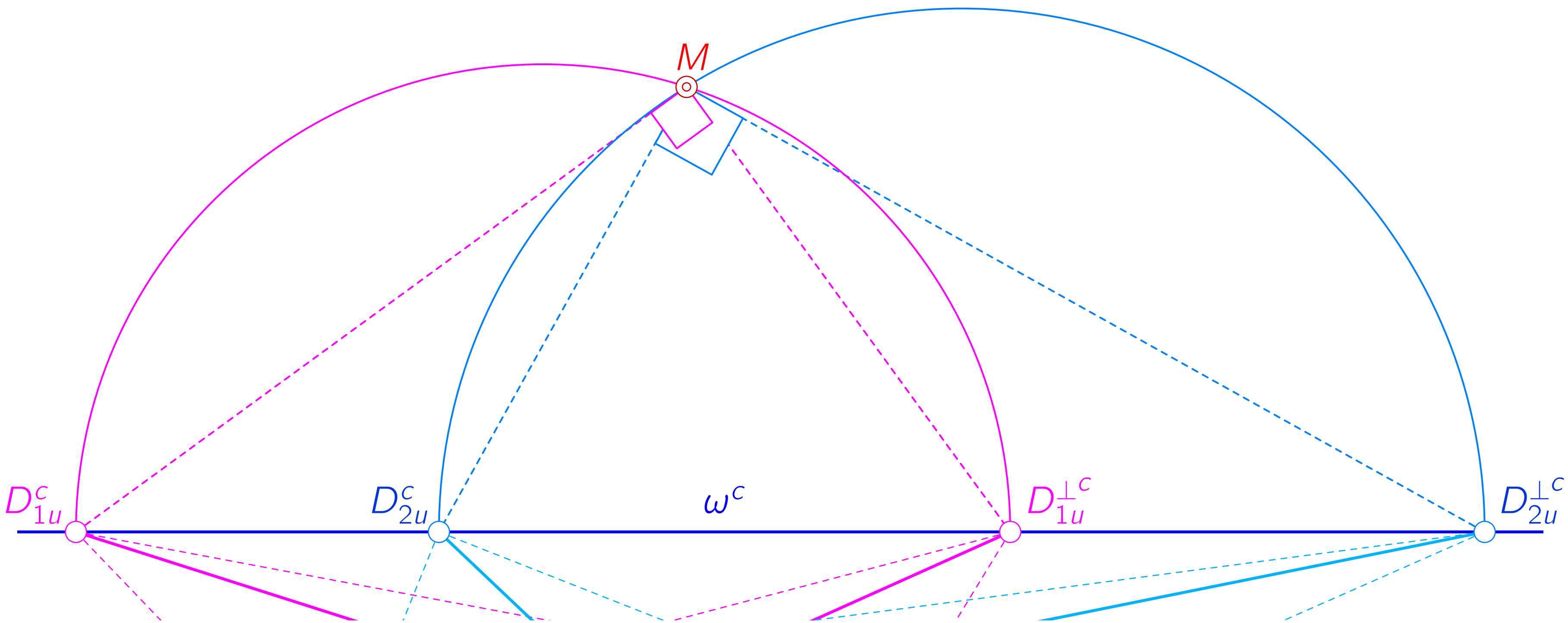
Conjugate diameters of a circle are orthogonal.

Orthogonality / diameters not preserved.

Persp. images of pairs of ideal points of conjugate diameters are pairs of points in an elliptic involution on the vanishing line.

Conjugate diameters

[3,10]



The measurement point M for the rectification that maps a conic c^C to a circle is the Laguerre point M of the elliptic involution on the vanishing line.

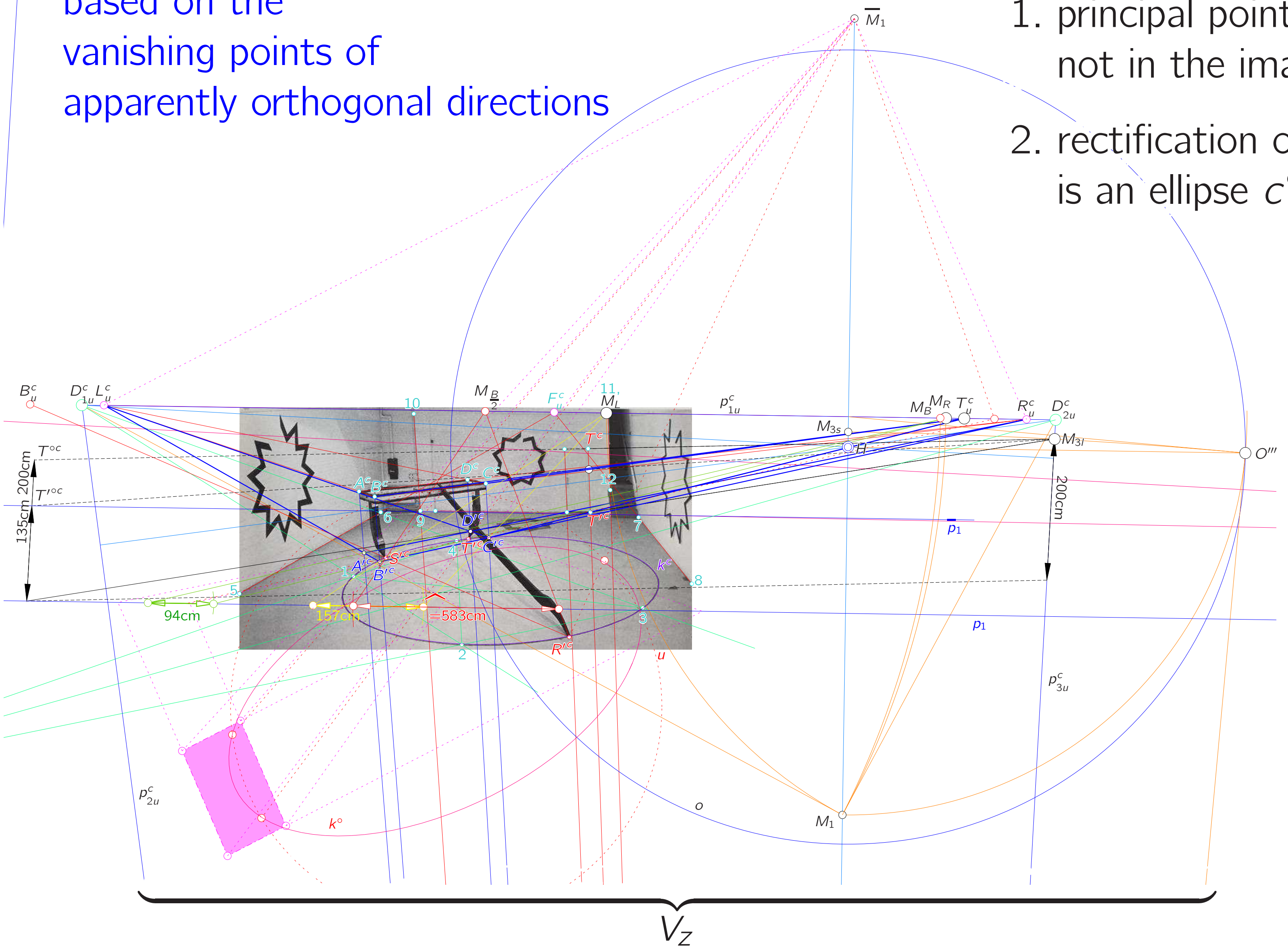
From M, conjugate pairs have to be seen at right angles. $\Longrightarrow$ M is a common point of Thales circles on conjuate vanishing points.

Solutions on both sides can serve as Measurement points.

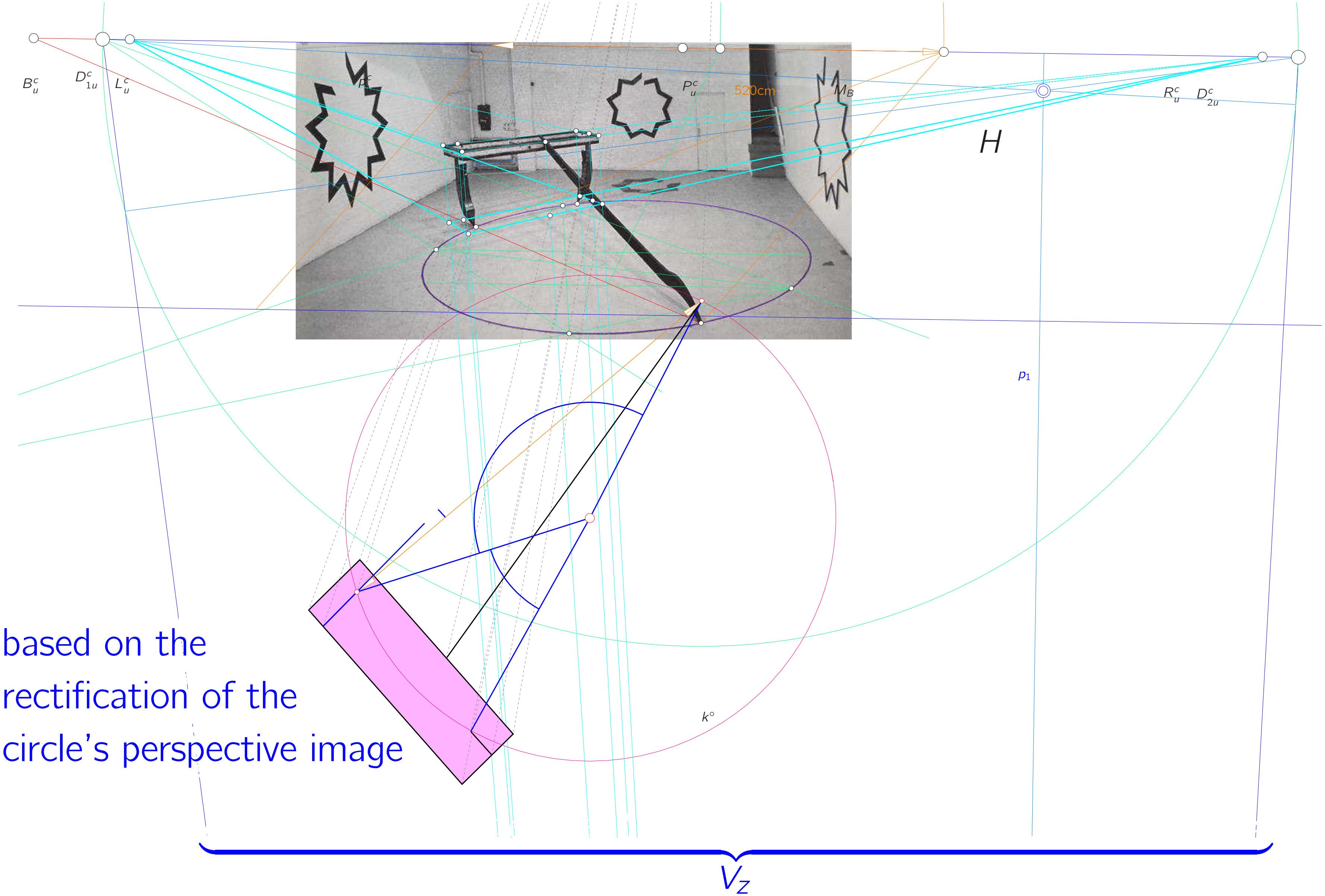
First attempt

based on the
vanishing points of
apparently orthogonal directions

1. principal point H
   not in the image center
2. rectification of c^C
   is an ellipse $c^\circ \neq$ circle

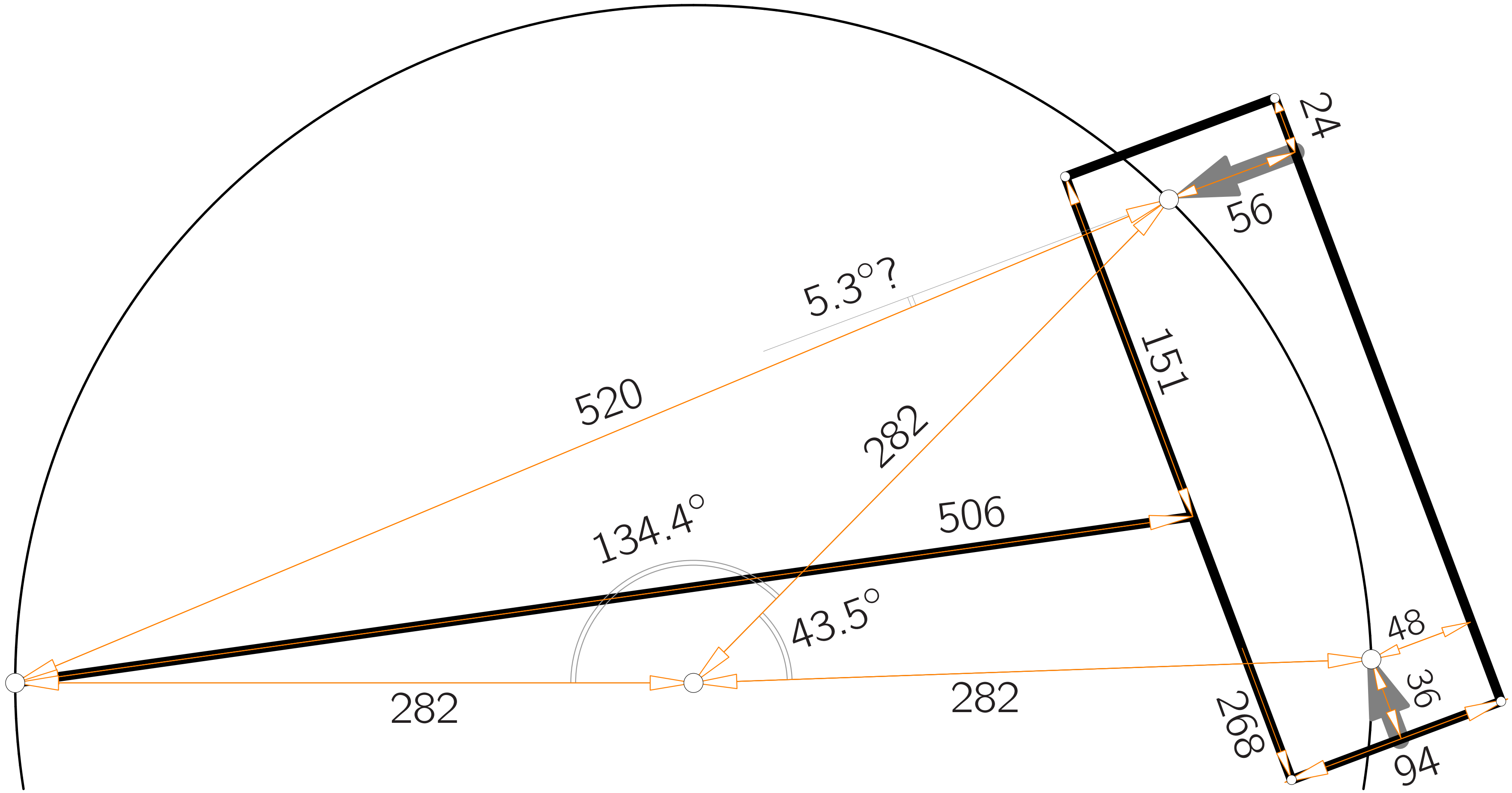


Second attempt



based on the rectification of the circle's perspective image

Top view of “Ramp and Hyphen”



Literature

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Thank You For Your Attention!