Subdivision schemes for ruled surfaces

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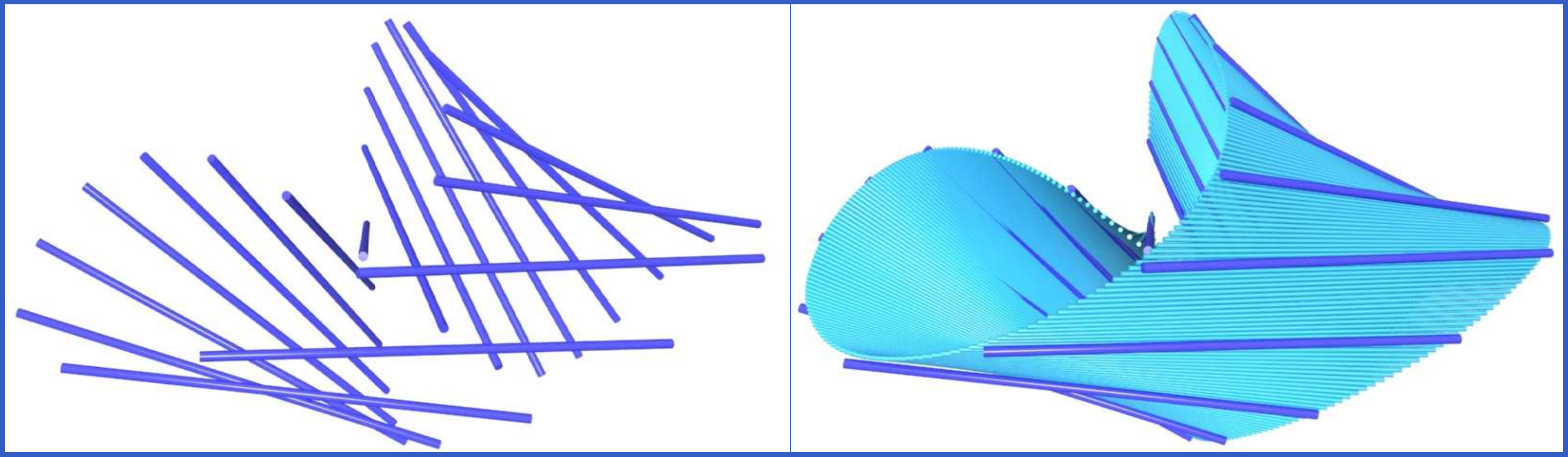
Aims & Motivation

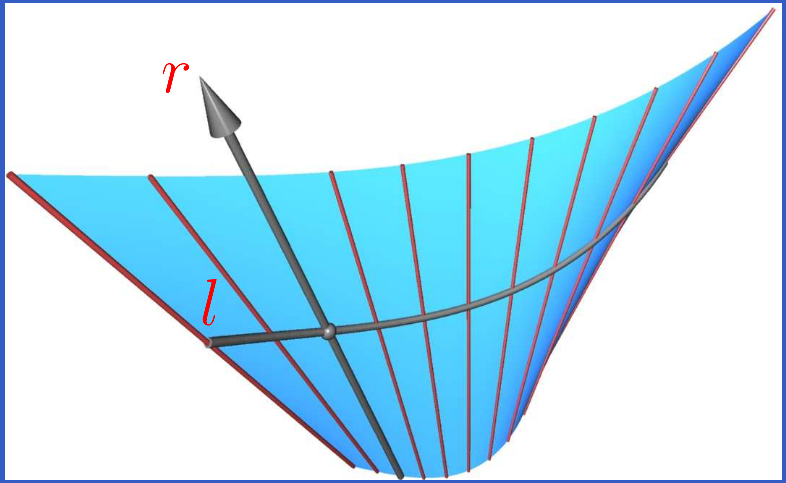
- approximation of ruled surfaces by discrete models
- define subdivision schemes for sets of lines
- circumvent parameterizations
- handle lines (not line segments), eliminate arbitrarily chosen directrices
- discrete models are more common in CAGD/CAD and computer graphics and have lots of applications

What are we going to do?

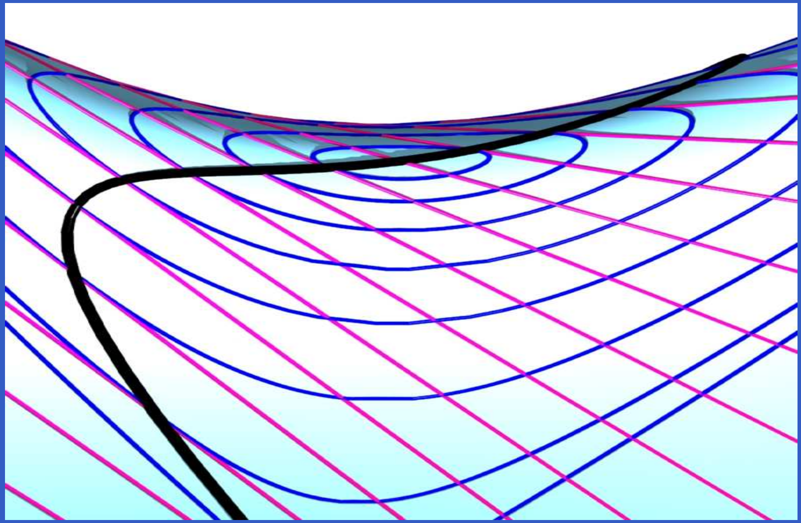
- given a coarse model of a ruled surface (finite set of lines)
- looking for a finer model (insert new lines)
- should lead to a pleasing (smooth) limit



Ruled surfaces



- smooth / discrete 1-param. family of lines (curve)
- directrix l, unit VF r parallel to rulings



- central curve $\mathbf{c}$:

$$c = l - \langle \dot{l}, \dot{r} \rangle / \langle \dot{r}, \dot{r} \rangle r$$

- locus of maximum Gaussian curvature

Klein model of line space

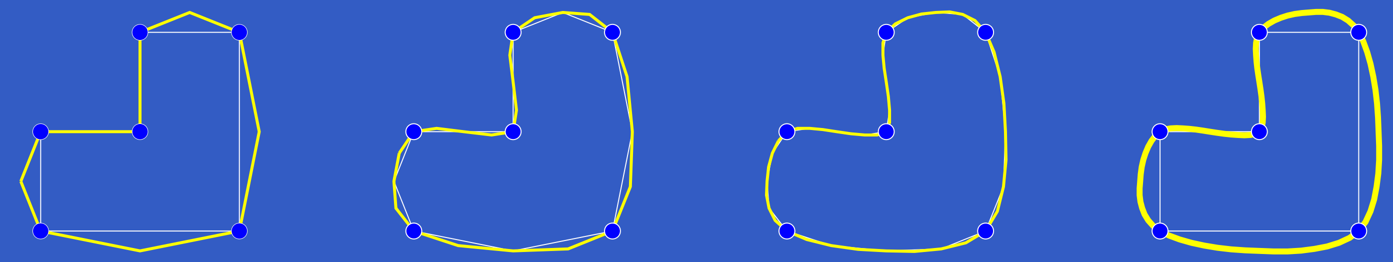
- lines $l + v \cdot r \to (r, l \times r) = (g, \overline{g})$
- Plücker coordinates $(g, \overline{g}) \in \mathbb{R}^{3+3}$ satisfy

$$M^4 : \ \langle g, g \rangle = 1, \quad \langle g, \overline{g} \rangle = 0$$

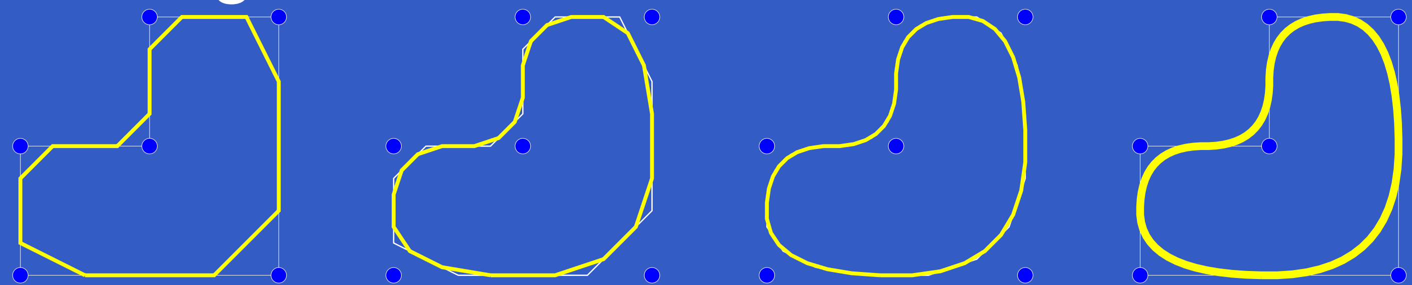
- ruled surfaces are curves in M^4

Subdivision schemes for curves

- interpolating scheme by DLG



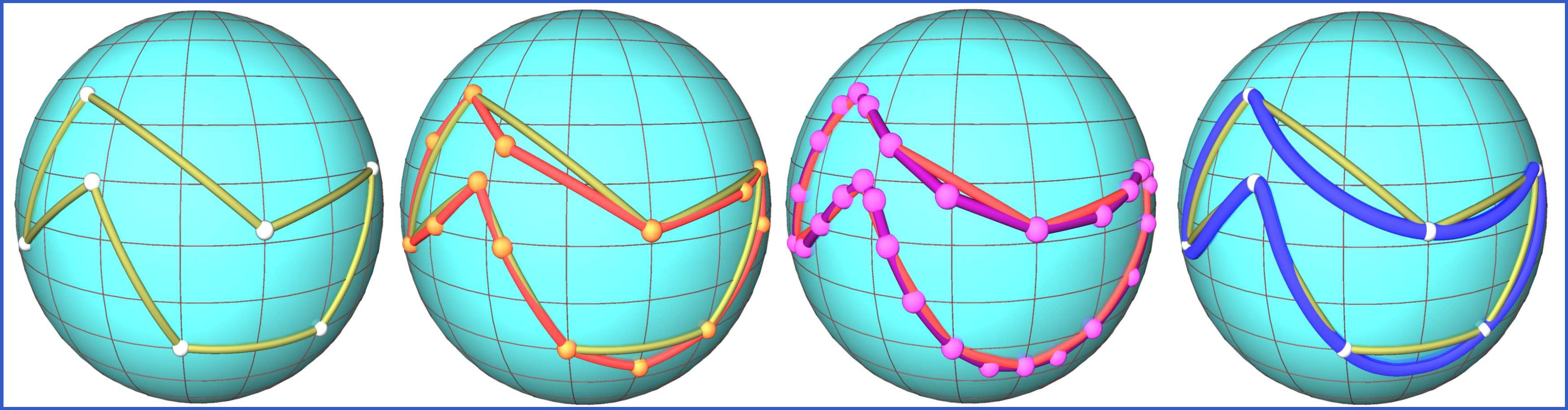
- approximating scheme: Chaikin's corner cutting



- known limits and precision (different mascs, ternary schemes, . . .)

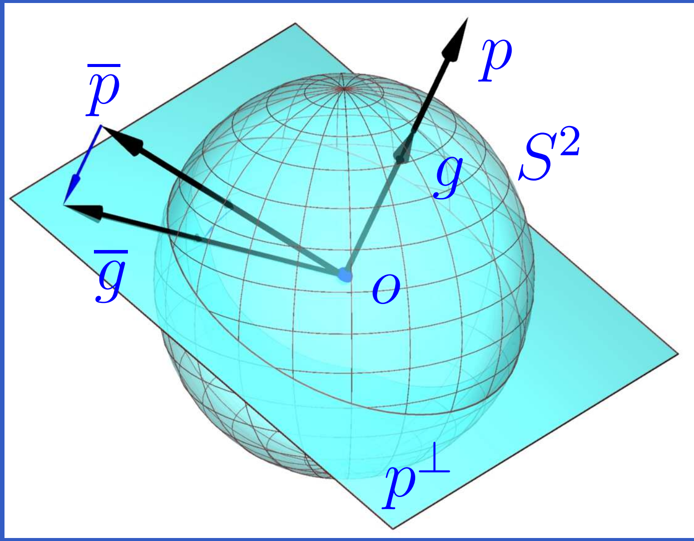
Subdivision schemes for curves

- originally for data in affine space
- generalized to data from arbitrary manifolds: geodesic subdivision, subdivison + projection



Algorithm 1: subdivision + projection

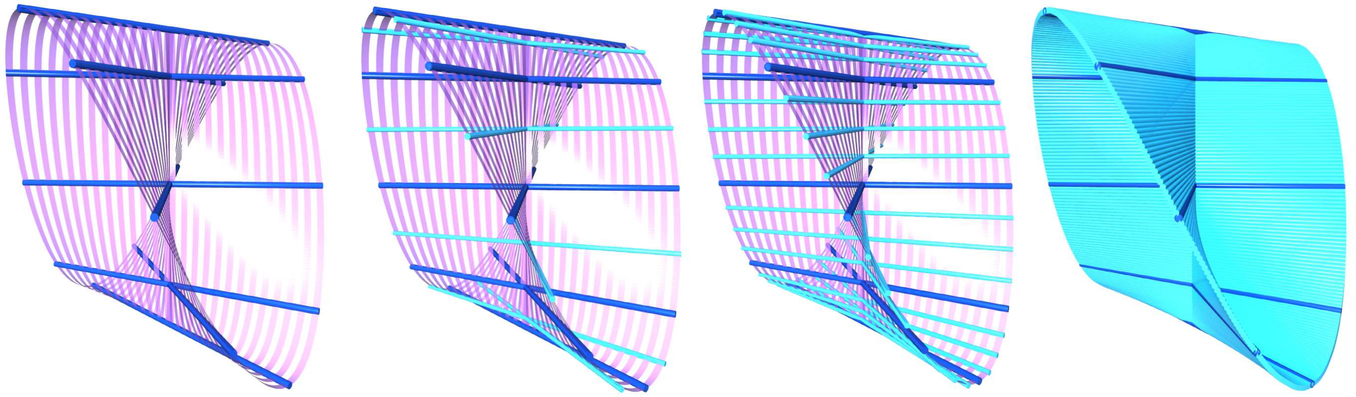
- discrete ruled surface = polygon in M^4 (vertices only)
- apply subdivision scheme to data
- project new vertices into M^4



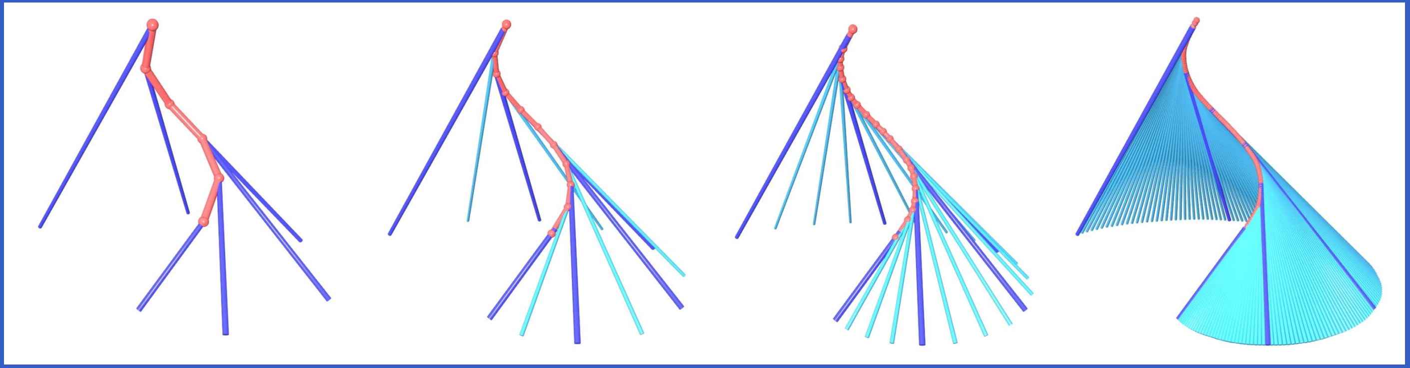
$$(g, \overline{g}) = \left(\frac{p}{\|p\|}, \frac{\overline{p}}{\|p\|} - \frac{p\langle p, \overline{p}\rangle}{\|p\|^3} \right)$$

Algorithm 1: examples

- reproducing Plücker’s conoid

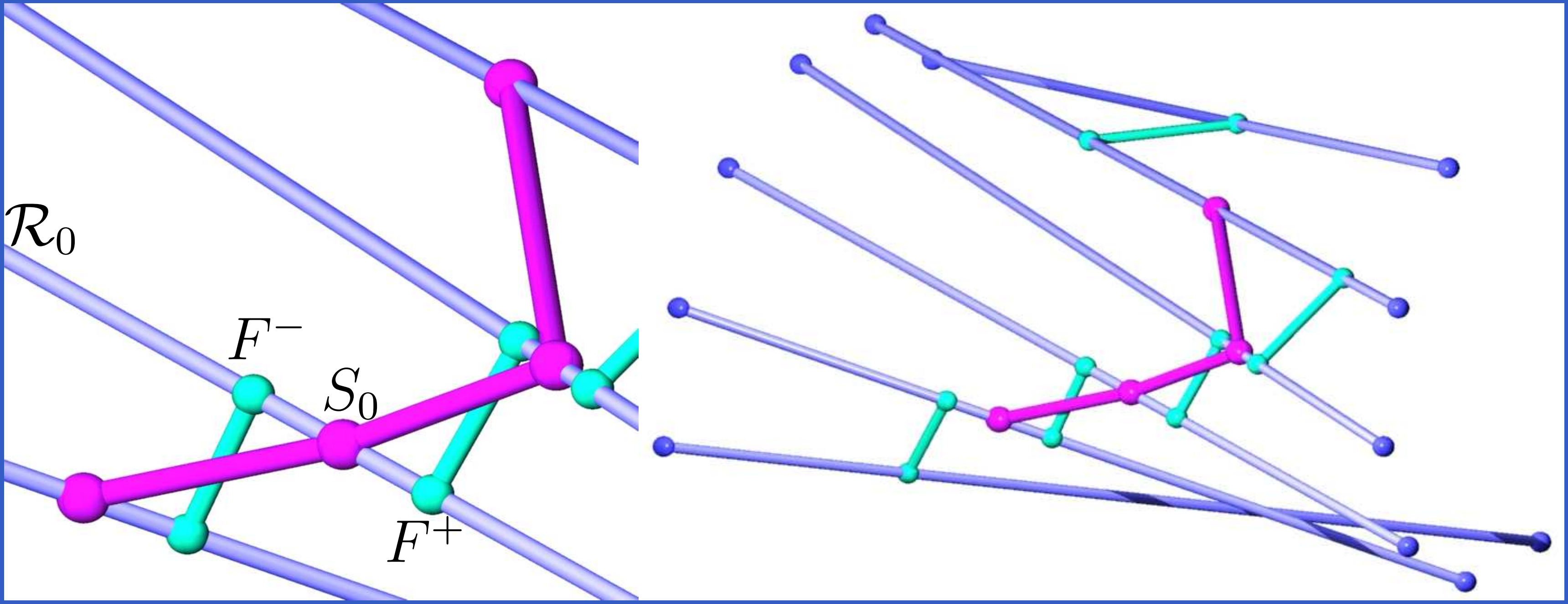


- algorithm handles torsal ruled surfaces properly



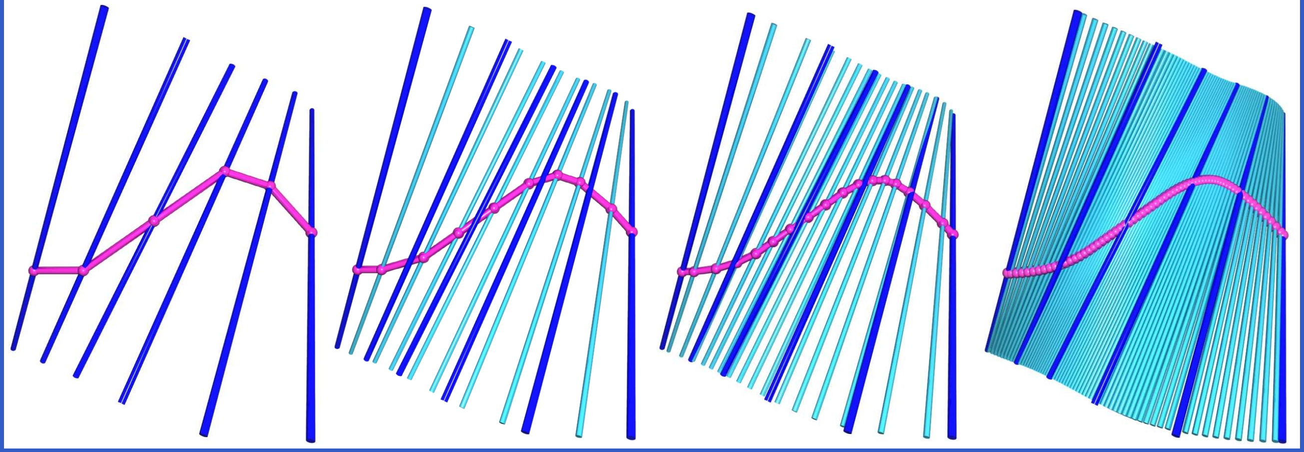
Algorithm 2: central curve + spherical image

- compute a discrete central curve
- refine a discrete version of the central curve
- refine discrete spherical image of the rulings

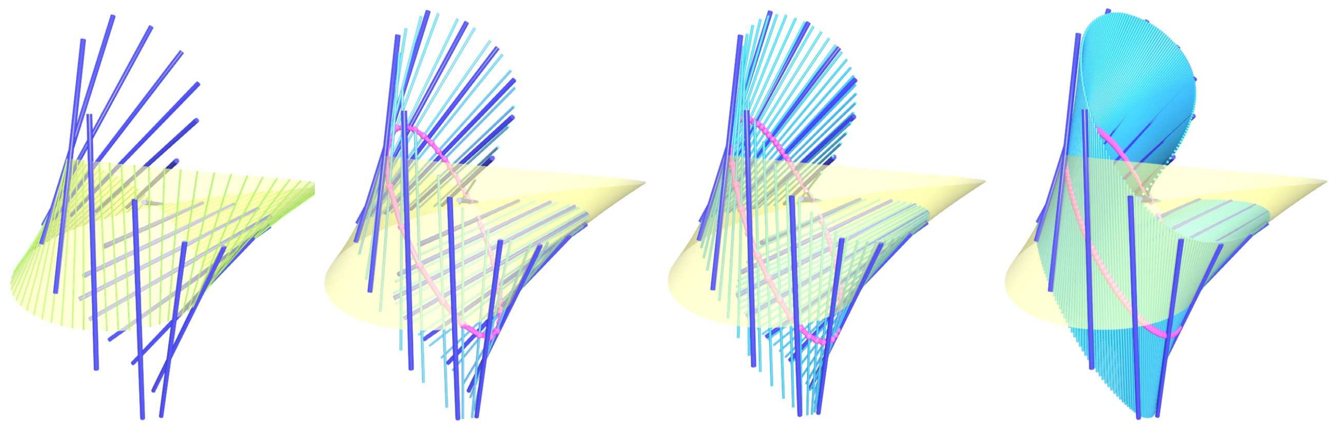


Algorithm 2: examples

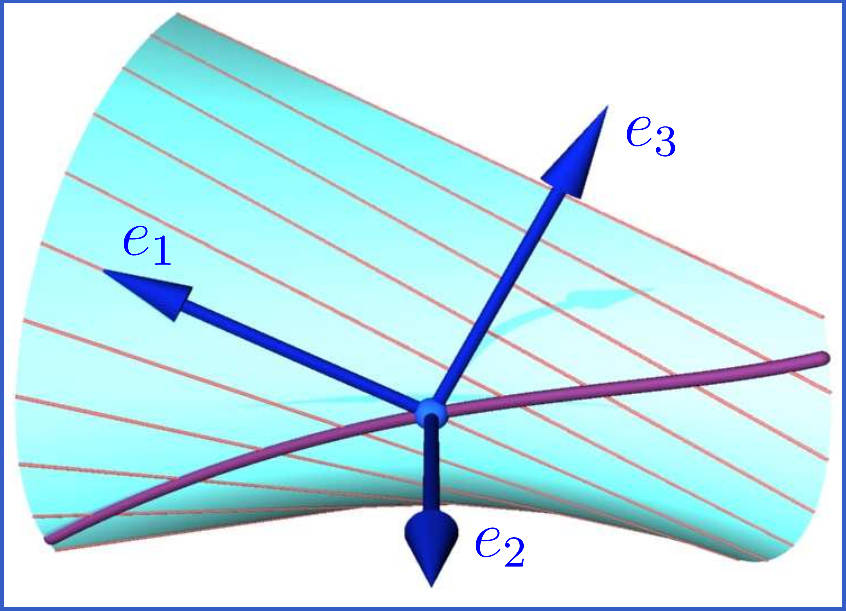
- approximating six arbitrarily given lines



- surface of Möbius type

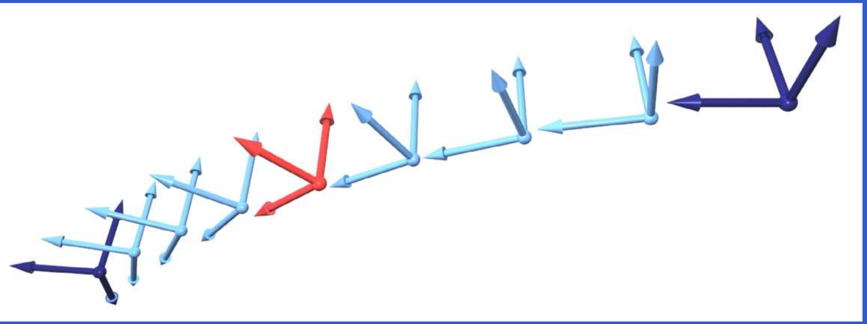


Algorithm 3: motion of the Sannia frame



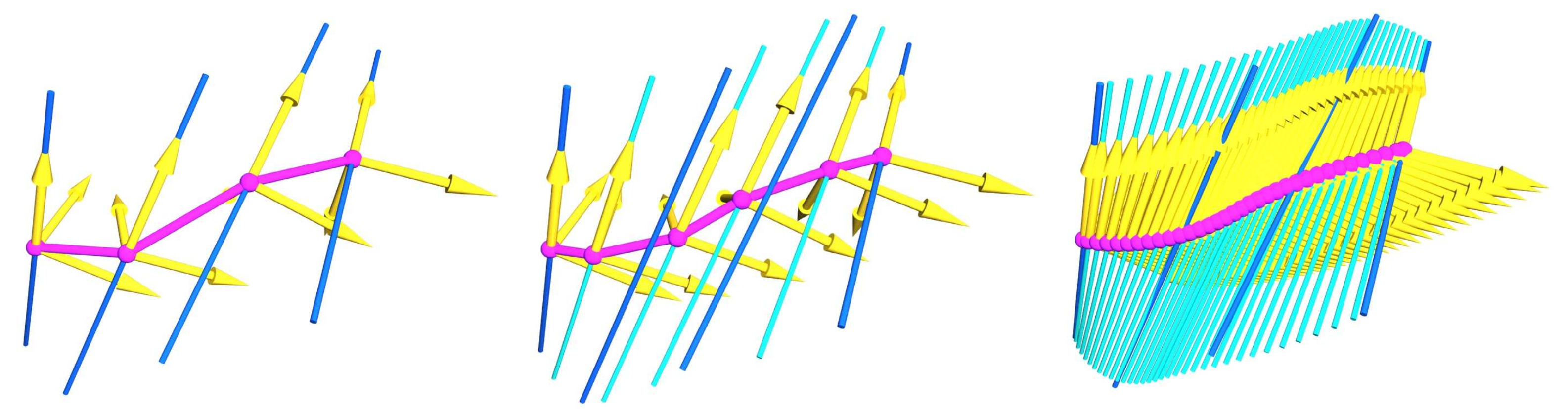
- Sannia frames define discrete 1-pm. motion
- refine Sannia motion by means of geodesic subdivision

- geodesics = helical motions
- computing intermediate positions

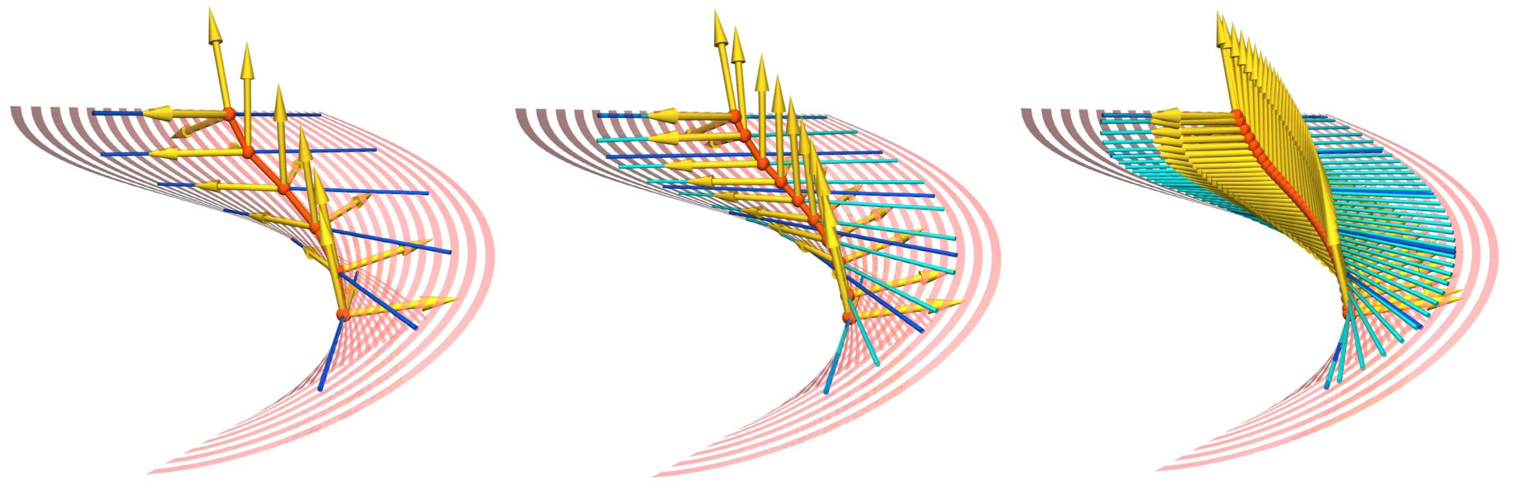


Algorithm 3: examples

- approximating four arbitrarily given lines



- data from a helical surface



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Thank you for your attention!